

**TEACHING AND LEARNING MODULE
ADDITIONAL MATHEMATICS FORM 5**

INTEGRATION

CHAPTER 3

NAME:

FORM :

Date received :

Date completed :

Marks of the Topical Test :

Prepared by :
Addational Mathematics Department
Sek Men Sains Muzaffar Syah Melaka

For Internal Circulations Only

Formulae

a) Area under a curve

$$= \int_a^b x \, dy$$

5 Volume generated

$$= \int_a^b \pi y^2 \, dx \quad \text{or}$$

$$= \int_a^b \pi x^2 \, dy$$

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Students will be able to:

1. Understand and use the concept of indefinite integral.
 - 1.1 Determine integrals by reversing differentiation.
 - 1.2 Determine integrals of ax^n , where a is a constant and n is an integer, $n \neq -1$.
 - 1.3 Determine integrals of algebraic expressions.
 - 1.4 Find constant of integration, c , in indefinite integrals.
 - 1.5 Determine equations of curves from functions of gradients.
 - 1.6 Determine by substitution the integrals of the form $(ax + b)^n$, where a and b are constants, n is an integer and $n \neq -1$.

1.1 Determining integrals by reversing differentiation.

- a) Integration is the inverse process of differentiation .
- b) The process of obtaining $\frac{dy}{dx}$ from y (a function of x) is known as differentiation. Hence, the process of obtaining y from $\frac{dy}{dx}$ is known as integration.
- c) Integration of y with respect to x , is denoted by $\int f(x)dx$
- d) If y is a function of x and $\frac{dy}{dx} = f'(x)$ then $\int f'(x)dx = y + c$ where c is arbitrary constant

1.2 Determining integrals of ax^n , where a is a constant and n is an integer, $n \neq -1$ and determining integrals of algebraic expressions.

Formula :

1. Integral of a constant $\int kdx = kx + c$ where c is a constant
2. Integral of ax^n (n integer, $n \neq -1$) is given by $\int ax^n dx = \frac{ax^{n+1}}{n+1} + c$

Example 2 . Integrate each of the following with respect to x

a) $\int 8dx =$	b) $\int -\frac{5}{6}dx =$	c) $\int x^6 dx$	d) $\int \frac{1}{x^5} dx$
e) $\int -\frac{3}{2x^4}$	f) $\int \frac{3}{\sqrt{x}} dx$	g) $\int \sqrt{x}$	h) $\int \frac{1}{x^2} dx$

Exercise 2 Integrate each of the following with respect to x

a) $\int 6x^3 dx$	b) $\int -x^5 dx$	c) $\int 5x^{-2} dx$	d) $\int \frac{1}{2x^2} dx$
e) $\int \frac{2}{x^2} dx$	f) $\int x^{-8} dx$	g) $\int \sqrt{x} dx$	h) $\int \frac{1}{2\sqrt{x}} dx$

Example 2 Integrate each of the following with respect to x

a) $\int (x^2 + 2x^3 - 6)dx$	b) $\int (9 - \frac{3}{x^2})dx$	c) $\int (x^2 + 2)^2 dx$	d) $\int \frac{(x+4)(x-4)}{x^2} dx$
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Exercise 2 : Integrate each of the following with respect to x

a) $\int (16x^7 + 2x^5 - 7x^3)dx$	b) $\int (7x - \frac{3}{x^6})dx$	c) $\int (\frac{4-x^2}{x^5})dx$	d) $\int (1-3x)(2+x)dx$
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1.4 Finding constant of integration, c , in indefinite integrals.

The value of the arbitrary constant in an integral can be found if one set corresponding values of the two variables are given. After integrating the function, substitute the given values and then evaluate the constant term

Example 3:

a) If $\frac{dy}{dx} = 2x + 3$, and $y = 3$ when $x = 2$, find y in terms of x . [$y = x^2 - 3x + 5$]	b) It is given that $\frac{dy}{dx} = kx^2 + 1$, where k is a constant When $x = 1$, $y = 6$ and when $x = -1$, $y = 2$ Find the value of y when $x = 2$. [Ans $y = x^3 + x + 4$, 14]
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Exercise 3

a) If $\frac{dy}{dx} = 4x^3 - 6x + 1$ and $y = 8$, when $x = 2$, Find y in terms of x . Find the value of y when [Ans $y = x^4 - 3x^2 + x + 2$, $y = 59$]	b) Given that $\frac{dy}{dx} = 2x + 6$ and $y = 10$ when $x = 2$, express y in terms of x [ans $y = x^2 + 6x - 6$]
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1.5 Determining equations of curves from functions of gradients.

Example 4:

<p>a) The gradient of a curve passing through the point (1,-1) is given by $\frac{dy}{dx} = x^2(2x+1)$. Find the equation of the curve . [Ans $y = \frac{x^4}{2} + \frac{x^3}{3} - \frac{11}{6}$]</p>	<p>b) The gradient function of a curve passing through the points is (0, 1) and (3, 1) is ax^2+2 . Find the value of a and the equation of the curve . [Ans $\frac{-2}{9}x^3 + 2x + 1$]</p>
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Exercise 4

<p>a) Find the equation of the curve that passes through the point (-1,0) and has the gradient function $\frac{dy}{dx} = 3x^2+2x+1$.. [Ans $y = x^3+x^2+x+9$]</p>	<p>b) The gradient function of a curve passing through the points is (-1, 8) and (3, 4) is $kx-3$, Find the value of k and the equation of the curve [Ans $y = x^2-3x+4$]</p>
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1.6 Determining by substitution the integrals of the form $(ax + b)^n$, where a and b are constants, n is an integer and $n \neq -1$

Integral of $(ax + b)^n$ is given by formula :

$$\int (ax + b)^n dx = \frac{(ax + b)^{n+1}}{a(n + 1)} + c$$

Example 5 : Integrate each of the following with respect to x

<p>(a) $\int (5x+3)^3 dx$</p>	<p>(b) $\int 3(6x-7)^5 dx$</p>	<p>(c) $\int \frac{1}{(2x+4)^4} dx$</p>	<p>d) $\int \frac{12}{(4-4x)^4} dx$</p>
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Exercise 5 : Integrate each of the following with respect to x

<p>1. $\int (5-3x)^4 dx$</p>	<p>2. $\int (4x-5)^{-3} dx$</p>	<p>3. $\int \frac{1}{(5-2x)^4} dx$</p>	<p>4. $\int (3-4x)^{-4} dx$</p>
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Students will be able to:

- 2. Understand and use the concept of definite integral.
- 2.1 Find definite integrals of algebraic expressions.
- 2.2 Find areas under curves as the limit of a sum of areas.
- 2.3 Determine areas under curve using formula.
- 2.4 Find volume of revolutions when region bounded by a curve is rotated completely about the a) x-axis b) y-axis as the limit of a sum of volumes
- 2.5 Determine volumes of revolutions using formula.

2.1 Finding definite integrals of algebraic expressions

$$\begin{aligned} \text{If } \int f(x)dx = F(x) + c \quad \text{then} \quad \int_a^b f(x)dx &= [F(x) + c]_a^b \\ &= [F(b) + c] - [F(a) + c] \\ &= F(b) - F(a) \end{aligned}$$

Example 6 : Evaluate

<p>1. $\int_{-1}^0 (4x^3 - x^2 + 1)dx$ [Jb -1/3]</p>	<p>2. $\int_{-1}^2 \frac{x^2 + 4}{x^2} dx$ [Jb 3]</p>	<p>3. $\int_0^2 (x^2 + 3)(2x - 1)dx$ [Jb 34/3]</p>
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Exercise 6 : Evaluate

<p>1. $\int_0^3 (3x^2 - x + 1)dx$ [Jb 51/2]</p>	<p>2. $\int_1^3 (2x - 1)(2x + 1)dx$ [Jb 98/3]</p>	<p>3. $\int_1^2 \frac{x^4 - 2}{x^2} dx$ [Jb 4/3]</p>
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Laws Of Definite Integrals

Laws	Example	Laws	Example
a) $\int_a^a f(x)dx = 0$	$\int_2^2 x^2 dx =$	d) $\int_a^b kf(x)dx = k \int_a^b f(x)dx$	$\int_1^2 2x dx =$
b) $\int_a^b f(x)dx = -\int_b^a f(x)dx$	Jika $\int_1^2 x dx = \frac{3}{2}$ maka $\int_2^1 x dx =$	e) $\int_a^b f(x)dx + \int_b^c f(x)dx = \int_a^c f(x)dx$ ie. $\int_0^1 3x dx + \int_1^3 3x dx =$	
c) $\int_a^b [f(x) \pm g(x)]dx = \int_a^b f(x)dx \pm \int_a^b g(x)dx$			

Example 7	Exercise 7
<p>1. Given that $\int_1^5 h(x)dx = 4$, evaluate</p> <p>(i) $\int_1^5 2h(x)dx =$</p> <p>(ii) $\int_1^5 \frac{hx}{3} dx =$</p> <p>(iii) $\int_1^5 [h(x) + 3]dx =$</p> <p>(iv) $\int_5^1 3h(x)dx =$</p> <p>(v) Find the value of k if $\int_1^5 [h(x) + kx]dx = 28$</p>	<p>2. If $\int_2^5 f(x)dx = 10$. evaluate</p> <p>(a) $\int_2^5 8f(x)dx =$</p> <p>(b) $\int_5^2 2f(x)dx =$</p> <p>(c) $\int_2^5 [f(x) - 1]dx =$</p> <p>(d) $\int_5^2 [3 - f(x)]dx =$</p> <p>(e) $\int_2^3 f(x)dx + \int_3^4 f(x)dx + \int_4^5 f(x)dx =$</p>
<p>2. If $\int_0^4 f(x)dx = 6$, find the value of $\int_0^4 \frac{f(x)+2}{2} dx$ [Ans 7]</p>	<p>2. Given that $y = \frac{3x-1}{x^2}$ and $\frac{dy}{dx} = 2h(x)$ where $h(x)$ is a function of x. Find the value of $\int_{-1}^1 h(x)dx$ [Ans 3]</p>

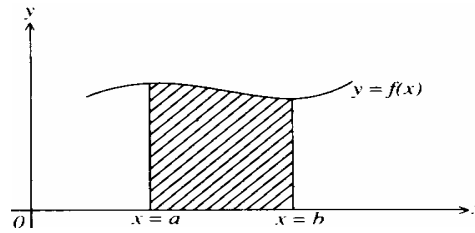
Ans 1 i) 8 ii) 4/3 iii) 16 iv) -12 v) k=2 2. a) 80 b) -20 c) 7 d) 1 e) 10

SPM QUESTIONS

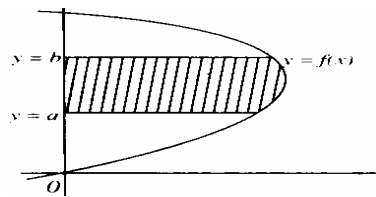
<p>1. SPM 1993 a) Find i) $\int \frac{(4-x)(4+x)}{x^2} dx$</p>	<p>ii) $\int \frac{18}{(3x-5)^3} dx$</p>	<p>(b) Given that $\int_0^3 f(x) dx = 8$ find the value of $\int_0^3 \frac{f(x)+2}{2} dx$</p>
<p>2.SPM 1994 Find $\int (2x+7)^3 dx$</p>	<p>3. SPM 95 Evaluate a) $\int_1^2 \frac{2(x-3)(x+3)}{3x^2} dx$</p>	<p>b) Given that $y = \frac{2x-1}{x^2}$ and $\frac{dy}{dx} = 2g(x)$ where $g(x)$ is a function of x. Find the value $\int_{-1}^1 g(x) dx$</p>
<p>4. SPM 1996 a) Given $\int_0^4 f(x) dx = 5$ and $\int_1^3 g(x) dx = 6$ Find the value of a) $\int_0^4 2f(x) dx + \int_3^1 g(x) dx$</p>	<p>b) k if $\int_1^3 [g(x) - kx] dx = 14$</p>	<p>5. SPM 1997 If $\int_1^3 f(x) dx = 6$, find the value of $\int_1^3 [2f(x) - 5] dx$</p>
<p>6. SPM 98 (a) Find $\int x^2(2 + \frac{1}{x^4}) dx$</p>	<p>(b) Find the equation of a curve that has the gradient function $(2x+1)^3$ and passes through the point $(\frac{1}{2}, -3)$</p>	<p>7. SPM 1999 Given $\frac{d^2y}{dx^2} = 4x^3 + 1$. When $x = -1$ $y = \frac{1}{2}$ and $\frac{dy}{dx} = 3$. Express y in terms of x</p>
<p>SPM 2000 Given $\int_1^5 f(x) dx = 4$ find the value of a) $\int_1^5 2f(x) dx$ b) k if $\int_1^2 f(x) dx + \int_2^5 [f(x) + kx] dx = 11$</p>	<p>SPM 2001 Answer 1.i) $\frac{-16}{x} - x + c$ ii) $\frac{3}{(3x-5)^2} + c$ a) 7 2. $\frac{(2x+7)^4}{8}$, 3. $7/3$ b) 2 3. b) 4 b) $k = -2$ 5. 2 6. 2 a) $\frac{2x^3}{3} - \frac{1}{x} + c$ b) $y = (2x+1)^4 / 8$ SPM 1999 $y = \frac{x^5}{5} + \frac{x^4}{2} + 3x + \frac{16}{5}$</p>	

- 2.2 Finding areas under curves as the limit of a sum of areas.
 2.3 Determining areas under curve using formula.

1 The shaded area between the curve $y=f(x)$, $x=a$, $x=b$ and the x -axis as shown is given by $\int_{x=a}^{x=b} y dx$

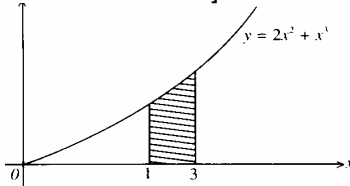


2. The shaded area between the curve $y=f(x)$, $y=a$, $y=b$ and y -axis as shown is given by $\int_{y=a}^{y=b} x dy$

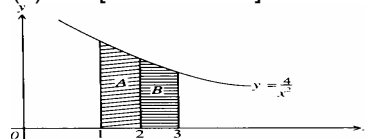


Example 7 Area between a curve and the x-axis

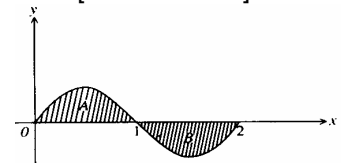
1 Calculate the area of the shaded region in the diagram below. [Ans $37 \frac{1}{3} \text{ unit}^2$]



Find the area of the shaded region
 (a) A [Ans 2 unit^2]
 (b) B [Ans $\frac{2}{3} \text{ unit}^2$]



4 Find the area enclosed by the curve $y = x(x-1)(x-2)$, lines $x=0, x=2$ and axis-x [Ans $\frac{1}{2} \text{ unit}^2$]



Exercise 7 :

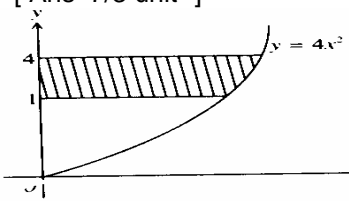
1. Find the area enclosed by the curve $y = 2x(3-x)$ and x -axis [Ans 9 unit^2]

2. Find the area enclosed by the curve $y = x^2 - 8x + 16$, x -axis lines $x=1$ and $x=6$ [Ans $11 \frac{2}{3}$]

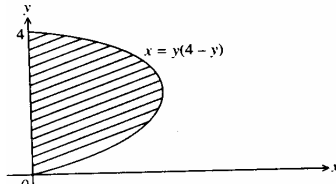
3. Find the area enclosed by the curve $y = x(x-1)(x-4)$ and x -axis [Ans $\frac{71}{6}$]

Example 8 : Area between a curve and the y – axis

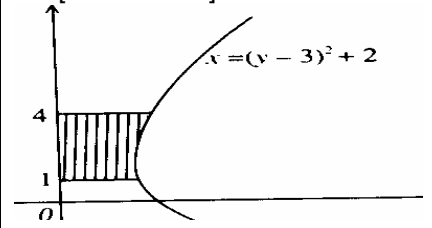
1. Find the area bounded by the curve $y = 4x^2$, lines $y = 4, y=1$, and y-axis .
[Ans $7/3$ unit²]



2. Find the area of the shaded region [Ans $10 \frac{2}{3}$ unit²]



3. Find the area of the shaded region [Ans 9 unit²]

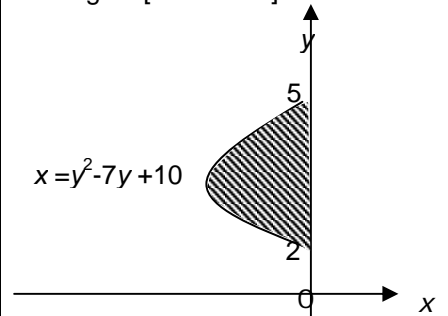


Exercise 8

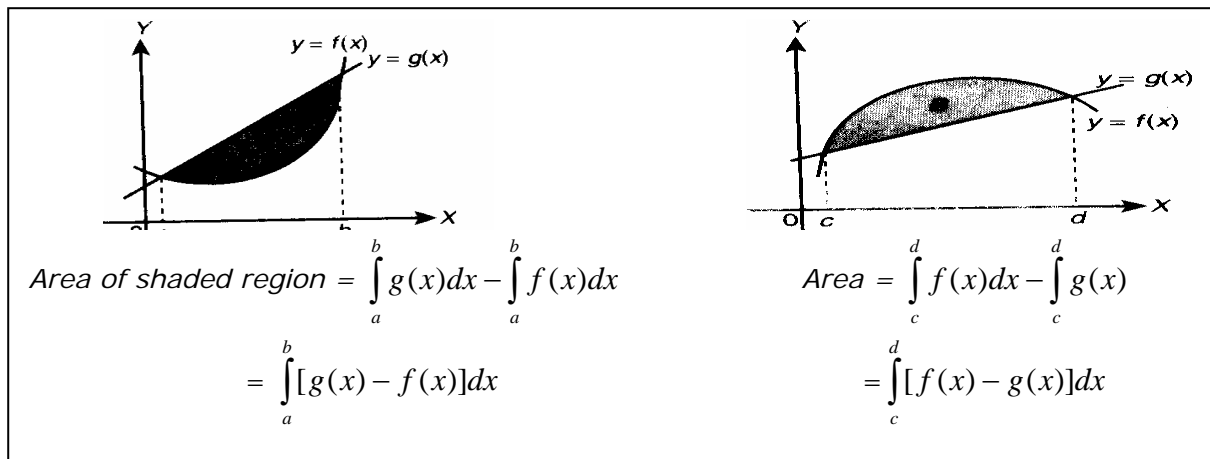
1. Find the area bounded by the curve $y = 4x$, lines $y=1, y=5$ and y – axis
[Ans 3unit²]

2. Find the area bounded by the curve $y^2=x$, lines $y = - 2, y=3$ and y – axis [Jb Ans $2/3$]

2. Find the area of the shaded region [Ans $4 \frac{1}{2}$]

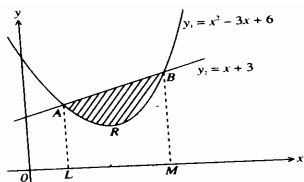


Area bounded by the curve $y = f(x)$ and straight line $y = g(x)$

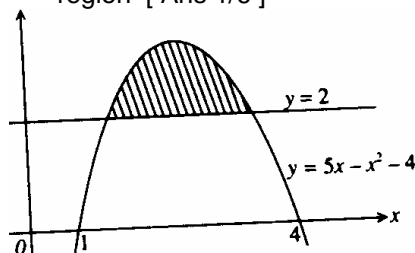


Example 9 :

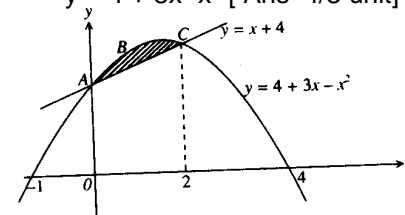
1. Diagram below shows a part of the curve $y_1 = x^2 - 3x + 6$ and line $y_2 = x + 3$
 (a) Find coordinate of L and M
 (b) Area of a shaded region
 [(1,4) (3,6) 4/3 unit]



2 Find the area of the shaded region [Ans 1/6]



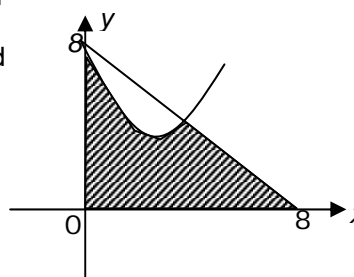
3 Find the area bounded by the line $y = x + 4$ and the curve $y = 4 + 3x - x^2$ [Ans 4/3 unit]



Exercise 9 :

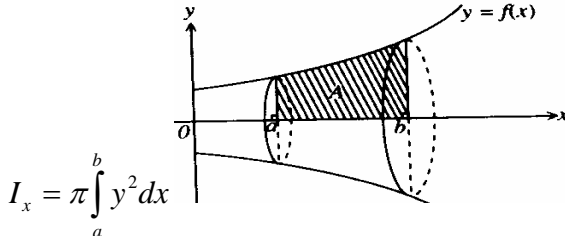
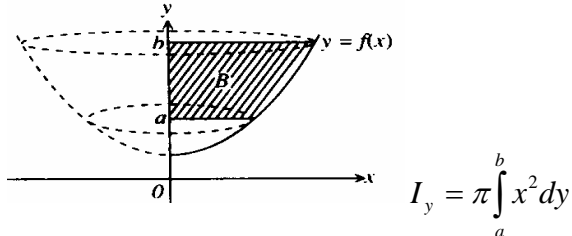
1. Find the area bounded by the curve $y = x(8-x)$ and the line $y = 3x$
 [Ans 20 5/6]

2. Diagram shows graphs of $y = x^2 - 3x + 8$ and $y = 8 - x$
 Find the area of the shaded region [30 2/3]

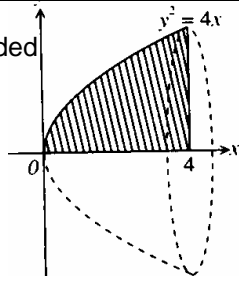
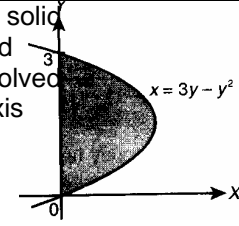


- 2.4 Finding volume of revolutions when region bounded by a curve is rotated completely about the
 a) x-axis b) y-axis as the limit of a sum of volumes
 2.5 Determining volumes of revolutions using formula

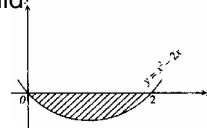
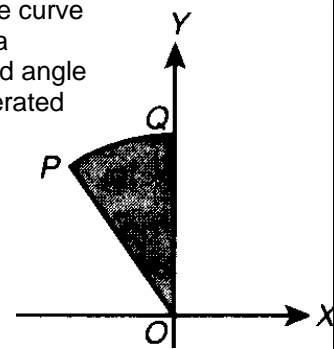
Volumes of solid of revolution

<p>Notes</p> <p>The volume of revolution V, generated when the area under a curve $y = f(x)$ by x-axis from $x = a$ to $x = b$ is rotated about the x-axis is</p>  $I_x = \pi \int_a^b y^2 dx$	<p>The volume of revolution V, generated when the area under a curve $y = f(x)$ by x-axis from $x = a$ to $x = b$ is rotated about the y-axis is</p>  $I_y = \pi \int_a^b x^2 dy$
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Example 10 : Finding Volume of solid of revolutions

<p>Find the volume of the solid of Revolution when the area bounded by the parabola $y^2 = 4x$ and line $x=4$, rotated about x - axis. through 360° [Ans 32π]</p> 	<p>Calculate the volume of the solid Generated when the shaded region in the diagram is revolved through 360° about the y-axis</p> <p>[Jb $\frac{81}{10}\pi$]</p> 
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Exercise 10

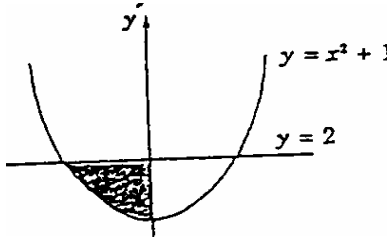
<p>1. Calculate the volume of the solid generated when the shaded region in the diagram is revolved through 360° about the x-axis</p> <p>{Jb $1\frac{1}{15}\pi$}</p> 	<p>2. In the diagram PQ is a part of the curve $x^2 + y^2 - 4 = 0$. Given that OPQ is a sector of a circle with centre O and angle $POQ = 30^\circ$. Find the volume generated when the sector OPQ revolved through 360° about the y-axis</p> <p>[Jb 2.245]</p> 
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SPM Question

SPM 93

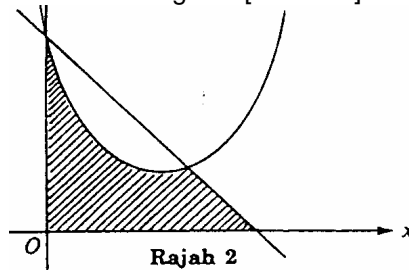
Calculate the volume of the solid generated when the shaded region in the diagram is revolved through 360° about the y-axis

[Ans $\frac{1}{2}\pi$]



SPM94

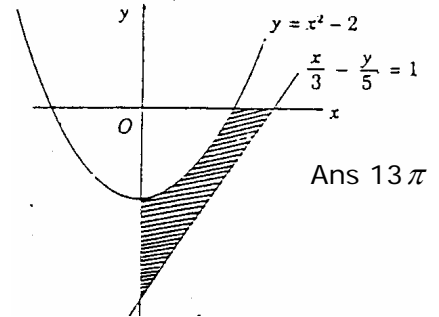
Rajah 2 shows the graph of $y = x^2 - 4x + 7$ and $y = 7 - x$. Calculate the shaded region [ans 20]



SPM98

Diagram below shows the graph of $y = x^2 - 2$ and straight line $\frac{x}{3} - \frac{y}{5} = 1$

Calculate the volume of the solid generated when the shaded region in the diagram is revolved through 360° about the y-axis

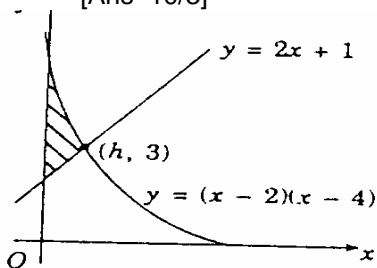


SM 97(a)

The diagram below shows the line $y = 2x + 1$ and the curve $y = (x-2)(x-4)$ that intersect at point $(h, 3)$

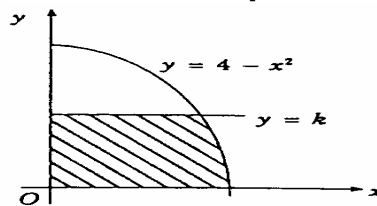
Find

- (i) the value of h [jb 1]
- (ii) area of the shaded region [Ans $10/3$]



SPM97(b)

The diagram below shows the shaded region which is bounded by the curve $y = 4 - x^2$, $y = k$ and y-axis. When the shaded region is revolved through 360° about y-axis, the volume generated is 6π unit³. Find the value of k [Ans $k = 2$]



SPM96

The diagram below shows a curve $y = x(x-1)(x+3)$. Find the area which is bounded by the curve, line $x = -2$, $x = 1$ and x-axis. [Jb 9]

