

module module TEACHING & LEARNING

ADDITIONAL MATHEMATICS FORM 5 LINEAR LAW

CHAPTER 2

NAME:

FORM :

Date received :

Date completed

Marks of the Topical Test :

Prepared by :
Additional Mathematics Department
Sek Men Sains Muzaffar Syah Melaka

For Internal Circulations Only

**L
E
A
R
N
I
N
G** **O
U
T
C
O
M
E
S**

Students will be able to:

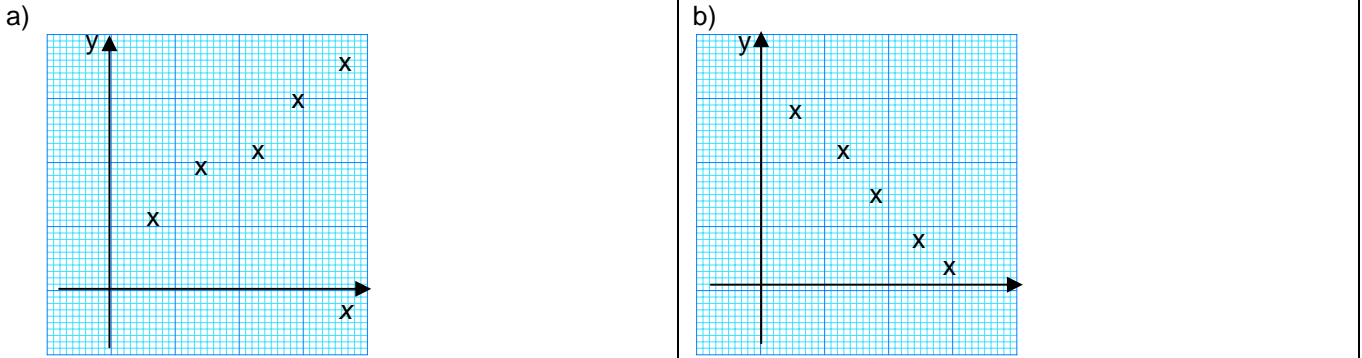
1. Understand and use the concept of lines of best fit.
- 1.1 Draw lines of best fit by inspection of given data.
- 1.2 Write equation for lines of best fit..
- 1.3 Determine values of variables from:
 - a) lines of best fit;
 - b) equations of lines of best fit.

1.1 Drawing the lines of best fit by inspection of given data.

The Line of best fit is the line that has the following properties

- i) The straight line is drawn in such a way that it passes through as many points as possible
- ii) The number of points that do not lie on the straight line drawn should be more or less the same on both sides of the straight line

Example 1 . Draw a line of best fit for the graphs below



1.2 Writing equation for lines of best fit

The step to form the equation of the line of best fit for a set of data that consists of two variables x and y as follows

- a) Choose a suitable scale so that the graph drawn is as large as possible
- b) Plot y against x and label both axes
- c) Draw a line of best fit
- d) Construct a right-angle triangle such that two vertices are on the line of best fit to calculate the gradient m , of the straight line
- e) Determine the y -intercept of the straight line (ie. The value of c)
- f) The equation of the line of best fit is given by $y = mx + c$

Example 2 : Based on an experiment, the corresponding values of x and y are obtained in the following table

a)

x	10	20	30	40	50	60	70
y	16.5	20.0	23.5	27.5	31.5	35.0	38.5

- a) Plot the graph y against x and draw the line of best fit
- b) Hence, form the equation of the straight line

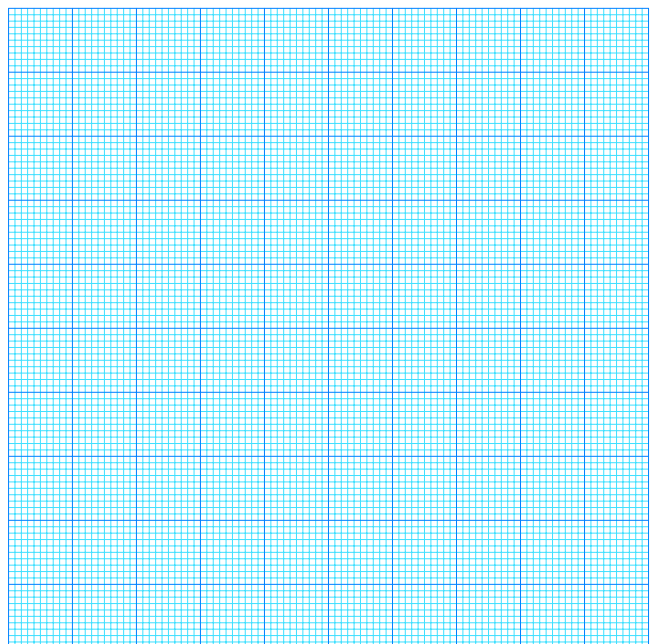
Solution :

b) From the graph,

The gradient m , =

the y -intercept =

The Equation of the line =



Example 2 : Based on an experiment, the corresponding values of x and y are obtained in the following table

a)

X	20	30	40	50	60	70
y	108	110.4	112.4	114.4	116.8	119.0

- c) Plot the graph y against x and draw the line of best fit
- d) Hence, form the equation of the straight line

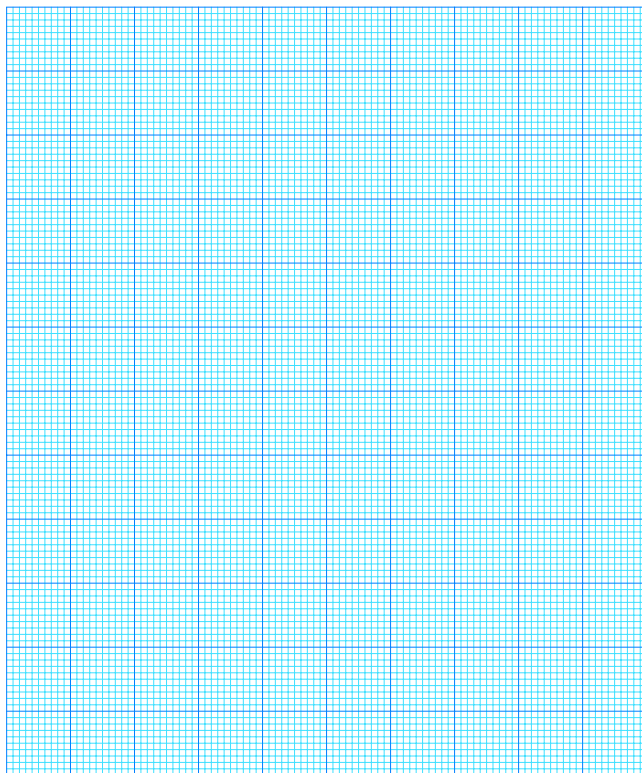
Solution :

b) From the graph,

The gradient m , =

the y-intercept =

The Equation of the line =



Homework Text Book Exercise 2.1 Page 35 No 1 - 3

1.3 Determining the values of variables from:

- a) lines of best fit
- b) equations of lines of best fit.

Example 3

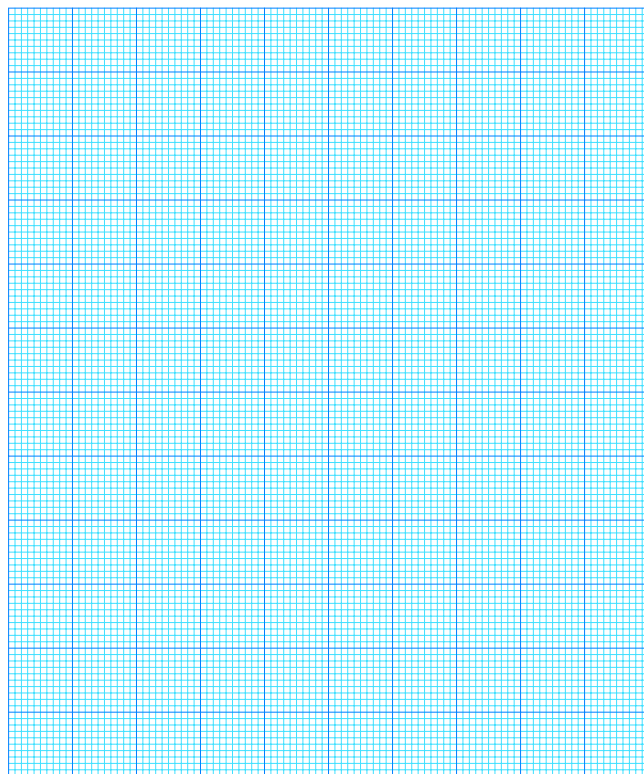
a The table below shows the data of two corresponding variables x and y that are related by the equation

$y = px + q$

x	1.8	2.5	3.6	4.9	5.3	6.4
y	4.8	6.2	8.4	11.2	11.8	13.8

- i) Plot the graph y against x and draw the line of best fit
- ii) From the graph, find the value of p and of q
- iii) Find the value of y when $x = 4.4$
- iv) Find the value of x when $y = 10.2$

Solution



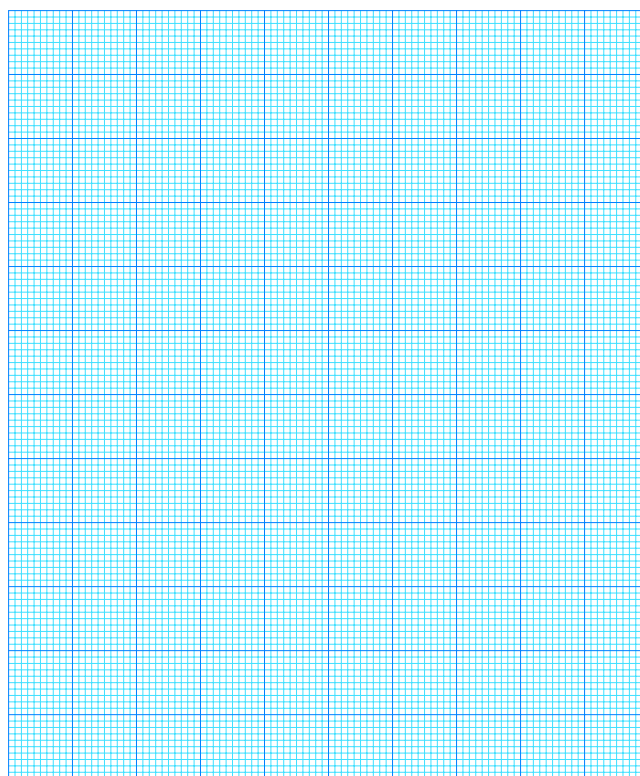
Exercise 3 Ans (ii) a) 28°C b) 32.5°C c) 5 s

a The table below shows the corresponding values of the temperature, $T^{\circ}\text{C}$ of the solution after time, t s in an experiment

t (s)	2	4	6	8	10
T ($^{\circ}\text{C}$)	29.0	29.9	31.0	32.1	33.0

- i) Draw the graph of T against t and draw the line of best fit
- ii) From the graph, find
- the initial temperature of the solution
 - the temperature of the solution after 9 seconds
 - the time taken to reach the temperature of 30.5°C

Solution



Homework Text Book Exercise 2.1 Page 35 No 4 - 11

LEARNING OBJECTIVES

Students will be able to:

- Apply linear law to non-linear relations..
 - Reduce non-linear relations to linear form.
 - Determine values of constants of non-linear relations given:
 - lines of best fit
 - data
 - Obtain information from:
 - lines of best fit
 - equations of lines of best fit.

- Applying the linear law to non-linear relations
 - Reduce non-linear relations to linear form.

Example 4 Reduce each of the following equations to the linear form Hence state the gradient and the y-intercept of the linear equations

no	Non-linear equation		Y-axis	X -axis	Gradient, m	y-intercept , c
1.	$y = \frac{k}{x^2}$	$y = k\left(\frac{1}{x^2}\right)$	y	$\frac{1}{x^2}$	k	0
2.	$y = ax^2 + bx$	$\frac{y}{x} = ax + b$	$\frac{y}{x}$	x	a	b
3.	$y = ax^3 + bx^2$	$\frac{y}{x^2} = ax + b$	$\frac{y}{x^2}$	x	a	b
4.	$xy = \frac{a}{x} + bx$	$y = \frac{a}{x^2} + b$	y	$\frac{1}{x^2}$	a	b
5.	$y = a\sqrt{x} + \frac{b}{\sqrt{x}}$	$y\sqrt{x} = ax + b$	$y\sqrt{x}$	x	a	b
6.	$\frac{a}{y} = \frac{b}{x} + 1$	$\frac{1}{y} = \frac{b}{a}\left(\frac{1}{x}\right) + \frac{1}{a}$	$\frac{1}{y}$	$\frac{1}{x}$	$\frac{b}{a}$	$\frac{1}{a}$
7.	$y = ab^x$	$\log_{10} y = (\log_{10} b)x + \log_{10} a$	$\log_{10} y$	x	$\log_{10} b$	$\log_{10} a$
8.	$y = ax^n$	$\log_{10} y = n\log_{10} x + \log_{10} a$	$\log_{10} y$	$\log_{10} x$	N	$\log_{10} a$

Exercise 4

Reduce each of the following equations to the linear form Hence state the gradient and the y-intercept of the linear equations

no	Non-linear equation	Y-axis	X -axis	Gradient, m	y-intercept , c
a	$y = 5x^2 + 3x$				
b	$y = p\sqrt{x} - \frac{q}{\sqrt{x}}$				
c	$y = ax^b$				
d	$ky = hx^2 + x$				
E	$xy = 10$				
f	$y = x + 2x^2$				
g	$y = pq^{-x}$				
h	$y = \frac{49}{a^2}(x+b)^2$				
i	$y = \frac{x^{3a}}{b}$				

Homework Text Book Exercise 2.2 Page 42 No 1 - 2

2.2 Determining the values of constants of non-linear relations given:

- a) lines of best fit b) data

Example 5

a The table below shows the data of two corresponding variables x and y that are related by the equation $y = a^{b+x}$ where a and b are Constance

x	1	2	3	4	5
y	2.83	5.66	11.31	22.63	45.25

- i) Reduce the non-linear equation $y = a^{b+x}$ to the linear form
 ii) Plot the graph of $\log y$ against x . Hence , draw the line of best fit
 iii) From your graph, find the value a and b
 [Answer $a = 2$, $b = 0.5$]

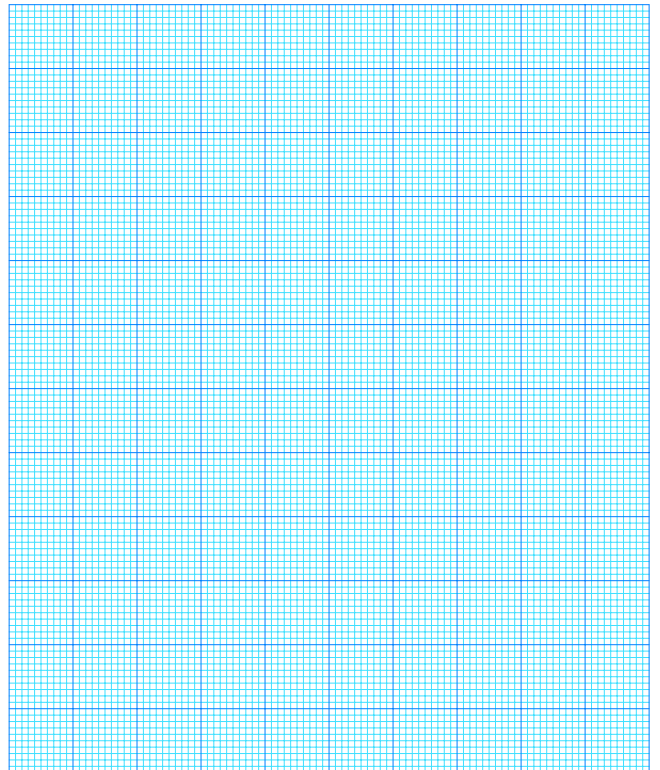
Solution

i) $y = a^{b+x}$

ii)

x	1	2	3	4	5
$\log y$					

iii)



Exercise 5

a The table below shows the data of two corresponding variables x and y that are related by the equation $y = ax^3 + bx^2$ where a and b are constants

x	0.5	1.0	1.5	2.0	2.5	3.0
y	0.31	2.05	6.19	14.00	26.30	45.00

i) Reduce the non-linear equation $y = ax^3 + bx^2$ to the linear form

ii) Plot the graph of $\log \frac{y}{x^2}$ against x .

iii) From your graph, find the value a and b
[Answer $a = 1.5$, $b = 0.47$]

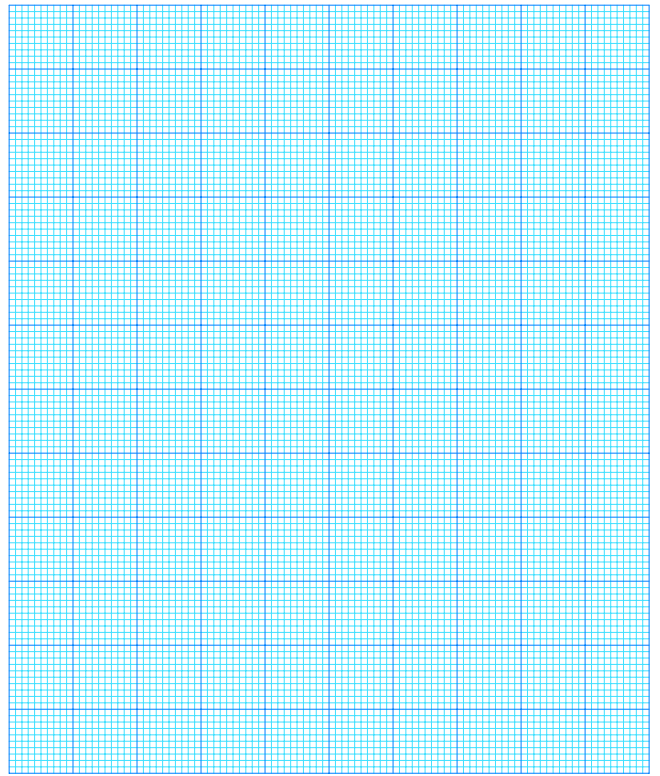
Solution

i) $y = ax^3 + bx^2$

ii)

x	0.5	1.0	1.5	2.0	2.5	3.0
$\frac{y}{x^2}$						

iii)

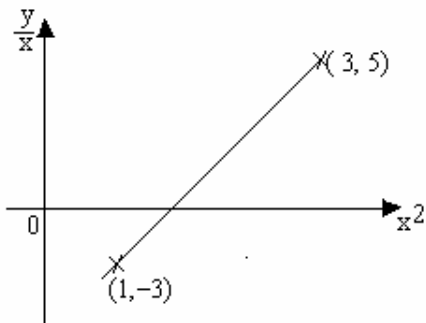


Homework Text Book Exercise 2.2 Page 42 No 3-8

2.3 Obtain information from: a) lines of best fit b) equations of lines of best fit

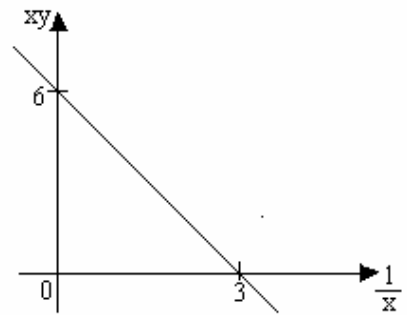
Example 6

a)



The variable x and y are related in such a way that when $\frac{y}{x}$ is plotted against x^2 , a straight-line that passes through the points $(1, -3)$ and $(3, 5)$ is obtained, as shown in the above figure. Express y in terms of x . Ans: $(y = 4x^2 - 7x)$

b)

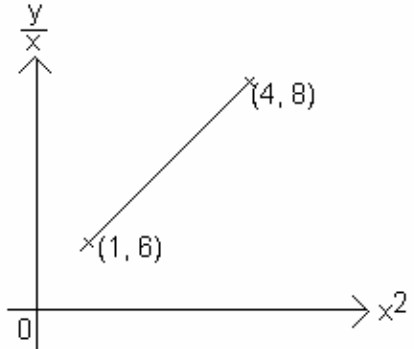


The variable x and y are related in such a way that when xy is plotted against $\frac{1}{x}$, a straight-line is obtained as shown in the figure above. Express y in terms of x .

Ans: $(y = -\frac{2}{x^2} + \frac{6}{x})$

Exercise 6

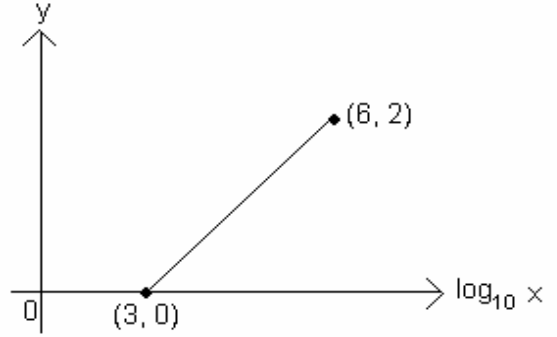
a)



The above figure shows part of a straight-line graph drawn to represent the equation $y = hx^3 + kx$, where h and k are constants. Find the value of h and of k .

[Ans $h = 4, k = 2$]

b)



The variables x and y are related in such a way that when y is plotted against $\log_{10} x$, a straight line that passes through the points $(3, 0)$ and $(6, 2)$ is obtained, as shown in the above figure. Calculate the value of y when $x = 1000$.

[Answer = 0]

Homework Text Book Exercise 2.2 Page 42 No 9 - 14

SPM Questions

SPM 2003

1)

x	1.5	2	3	4	5	6
y	3.09	3.76	5.12	6.32	7.48	8.55

The table above shows the results of an experiment carried out on the variables x and y . It is known that x and y are related by the equation $y^2 = px^n$, where p and n are constants.

(a) Plot the graph of $\log_{10}y$ against $\log_{10}x$, by using a scale of 2cm to 0.1 unit on both axes. Hence, draw the line of best fit.

(b) Use the graph from (a) to find the value of

(i) p
(ii) n

[Ans : (i) $p = 5.248$, (ii) $n = 1.438$]

SPM 2004

2

Length, l (cm)	10	20	30	40	50	60
Time of oscillations, T (s)	6.3	9	11	12.6	14.1	15

The table above shows the data obtained from an experiment on the motion of a simple pendulum where l (cm) is the length of the string and T (seconds) is the time of oscillations. One of the data for T is suspected to be wrongly recorded.

(a) Draw the graph of T^2 against l ,
(b) Mark \otimes on the graph for the point which is wrongly recorded. Then find the correct value of T .
(c) Using the graph obtained, find the value of k if T and l are related by the equation $\sqrt{l} = \frac{T}{k}$, where k is a constant.

[Ans : (b) $T = 15.5$, (c) $k = 2$]