# ADDITIONAL MATHEMATICS WORKSHOP
## FORM 5

# LINEAR PROGRAMMING

CHAPTER 10

NAME:………………………………………………….

FORM :…………………………………………….

Date received : ………………………………

Date completed ………………………….

Marks of the Topical Test : ……………………………..


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Introduction

Many people rank the development of linear programming among the most important scientific advances of the mid-twentieth century. Its seeds were sown during World War II when the military supplies and personnel had to be moved efficiently, and from that date its impact becomes extraordinary. In fact, a very major proportion of all scientific computation on computers is devoted to the use of linear programming and closely related techniques.

A *Linear Programming* problem is a special case of a *Mathematical Programming* problem. It uses a mathematical model to describe the problem of concern. The adjective “linear” means that all the mathematical functions in this model are required to be linear functions. The word “programming” is essentially a synonym for planning. Thus, linear programming involves identifying an *extreme* (i.e., minimum or maximum) point of a function **,** which furthermore satisfies a set of constraints,

Linear programming is widely used in industry, governmental organizations, ecological sciences, transportation and business organizations to minimize objectives functions, which can be production costs, numbers of employees to hire, or quantity of pollutants released, given a set of constraints such as availability of workers, of machines, or labors time.

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L e a r n i n g   O u t c o m e s

*Students will be able to:*

1. Understand and use the concept of graphs of linear inequalities.

1.1 Identify and shade the region on the graph that satisfies a linear inequality.

1.2 Find the linear inequality that defines a shaded region.

1.3 Shade region on the graph that satisfies several linear inequalities.

1.4 Find linear inequalities that define a shaded region.

1.1 Identify and shade the region on the graph that satisfies a linear inequality.

Note .

A line $y = mx + c$ divides a Cartesian plane into two regions. The region above the line including the line is $y \geq mx + c$ , while the region below the line including the line is $y \leq mx + c$.

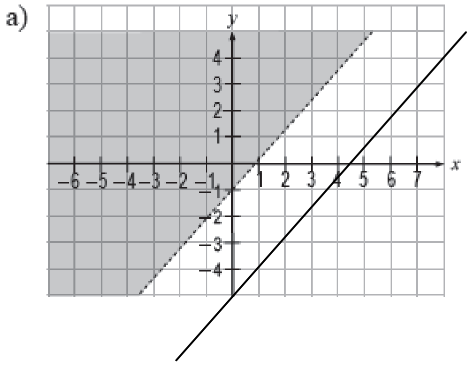
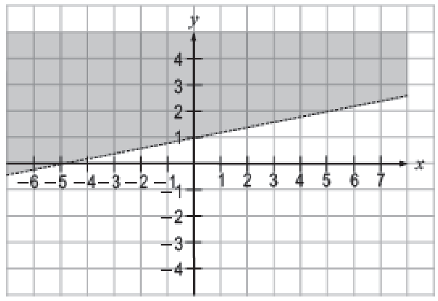
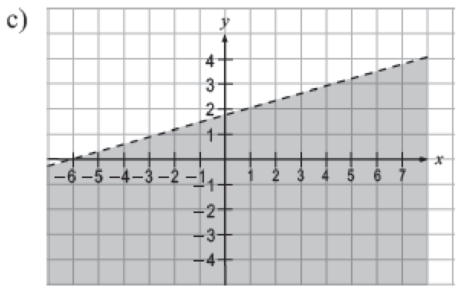
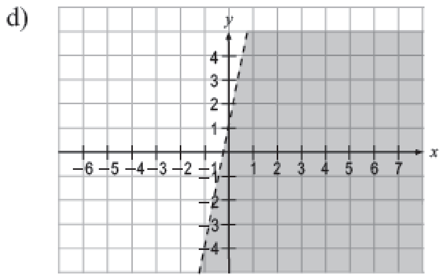
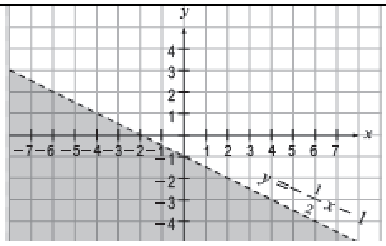
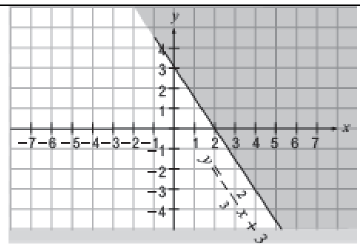
2. A dashed line is used when the region exclude the line
3. A region which satisfies a few inequalities is obtained by the intersection of the regions obtained by each inequality.
4. Linear Programming can be used to solve problems by using graphs. The following are the step
   (a) Determining the two variable x and y
   (b) Form a system of inequalities
   (c) Draw the graph for each inequality
   (d) Determine the region which satisfies all the inequalities
   (e) Form an optimum function k = ax + by
   (f) Select a suitable value of k and draw the line ax + by = k
   (g) With the use of ruler and set square, slide the line towards the region to obtain the required optimum function

Exercise 1 : Shade the region *R* which satisfy each of the following linear inequalities on separate graph.

| | | | |
|---|---|---|---|
| a) $x \geq 2$ | b) $y < -2$ | c) $y \geq 0$ | d) $x \geq 0$ |
| e) $y \geq -4$ | f) $y < 3$ | g) $y \geq x$ | h) $y \leq 2x-4$ |
| i) $x - y \leq 3$ | j) $x + 2y \leq 6$ | k) $5y \leq 2x + 10$ | l) $6x + y \geq 6$ |

**Example** 1.2 Finding the linear inequality that defines a shaded region

Determine the linear inequality represented by each of the shaded regions below.

| | |
|---|---|
|  | b)  |
| c)  |  |
| e)  | h)  |

## 1.3 Shading region on the graph that satisfies several linear inequalities

Note :

> The region that satisfies several inequalities is the intersection of all the region representing each inequality

Example 3; Sketch and shade the region R that satisfies each of the following system of linear inequalities

| | |
|---|---|
| a) $y \geq 1$ , $x \leq 3$ , $y \leq x + 1$ | b) $x \geq -2$ , $y \geq x$, $x + y \leq 1$ |

| | |
|---|---|
| c) $y \le 2x$, $2y \ge x$ , $x + y \le 4$ | d) $x+y \le 2$ , $y \ge x-1$ , $2x + y \ge 2$ |
| e) $x + y \le 3$ , $2y-3x \le 6$, $2y \ge x + 2$ | f) $2x + y \ge 4$, $x - 2y \le 2$ , $y \le 2$ |
| g) $x \ge 0$ , $y \ge 0$, $2y - x \le 6$, $5x + 3y \le 15$ | h) $2x + y \ge 4$ , $y \ge x-3$, $y < 3$ , $x < 4$ |
| i) $6x + 7y < 42$, $y \ge 1$, $3x + 2y > 6$, $y-x \le 3$ | j) $y \le 5$, $x < 4$, $y-x < 2$ , $2x + 5y \ge 10$ |

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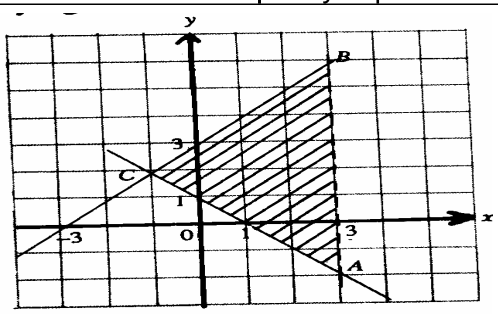
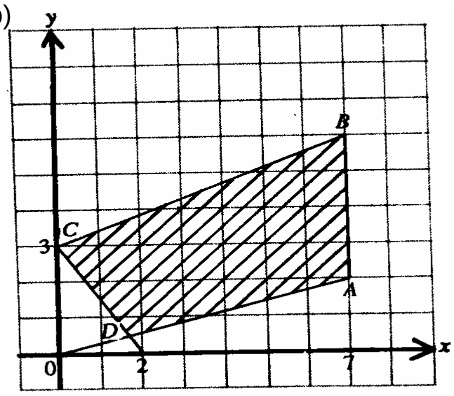
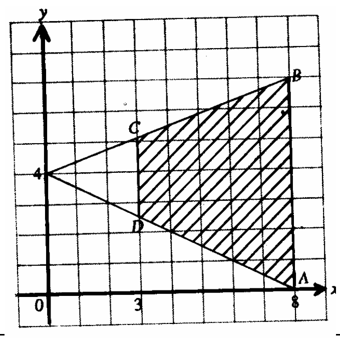
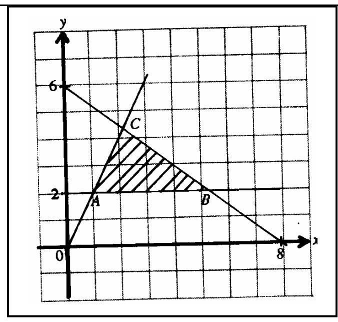
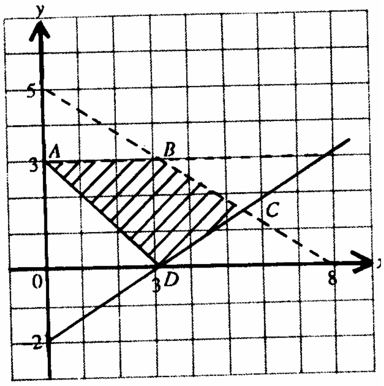
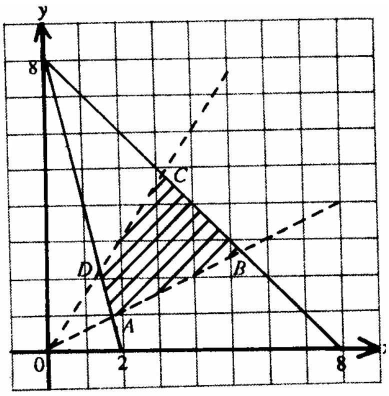
**Example** 1.2 Finding the linear inequality that defines a shaded region

Determine the linear inequality represented by each of the shaded regions below.

| | |
|---|---|
| a)   | b)   |
| c)   | d   |

| | |
|---|---|
| | |
| e)  | h)  |





**Learning Outcomes**

*Students will be able to:*

2. Understand and use the concept of linear programming..

2.1 Solve problems related to linear programming by:

a) writing linear inequalities and equations describing a situation.

b) shading the region of feasible solutions.

c) determining and drawing the objective function *ax* + *by* = *k* where *a, b* and *k* are constants.

d) determining graphically the optimum value of the objective function.

2.1 Writing linear inequalities and equations describing a situation

Example 4 : Write down an inequality based on a statement

| | | |
|---|---|---|
| a) *y* is greater than *x* | b) *y* is less than *x* | c) *y* is not more than *x* |
| d) *y* is not less than *x* | e) *y* is at least *p* times of *x* | f) *y* is at most *p* times of *x* |
| g) The total of *x* and y is not more than *p* | h) the total of *x* and *y* is at least *k* | i) the smallest value of y is *p* |
| j) the greater value of *y* is *q* | k) x exceeds two times of y by at least k | l) the ratio of *y* to *x* is *q* or more |

Example 5 Write down an inequality which satisfy the following constraints

| | |
|---|---|
| SPM 2003<br>Yahya has an allocation of RM 225 to buy *x* kg of prawns and *y* kg of fish. The total mass of the commodities is not less than 15 kg. The mass of prawns is at most three times that of fish. The price of 1 kg of prawns is RM 9 and the price of 1 kg of fish is RM 5 .Write down three inequalities that satisfy all of the above conditions | SPM 2004<br>A district education office intends to organise a course on the teaching of Mathematics and science in English. The course will be attended by x mathematics participants and y Science participants. The selection of participants is based on the following constraints<br>**I** : The total number of participants is at least 40<br>**II**: The number of science participants is at most twice that of Mathematics<br>**III**. The maximum allocation for the course is RM 7 200 . The expenditure for a Mathematics participant is RM 120 |

| | |
|---|---|
| | and for Science participant is RM 80 |
| SPM 2005<br>An institution offers two computer courses, *P* and *Q*. The number of participant for course *P* is *x* and for course *Q* is *y*. The enrolment of the participants is based on the following constraints<br>**I** : The total number of participants is not more than 100<br>**II**: The number of participants for course *Q* is not more than 4 times the number of participants for course *P*.<br>**III**: The number of participants for course *Q* must exceed the number of participants for course *P* by at least 5 | |

b) Shading the region of feasible solutions.

The regions that satisfies all the inequalities is the region of feasible solutions. This means that the possible solutions to the problem lie inside this region. We draw a graph to determine the feasible region.

Exercise 5

| SPM 2003 | SPM 2004 | SPM 2005 |
|---|---|---|
| By Using a scale of 2 cm to 5 kg for both axes, construct and shade the region R that satisfies all the conditions | By Using a scale of 2 cm to 10 participants on both axes, construct and shade the region R that satisfies all the conditions | By Using a scale of 2 cm to 10 participants on both axes, construct and shade the region R that satisfies all the conditions |

c) Determining and drawing the objective function $ax + by = k$ where *a, b* and *k* are constants.

d) Determining graphically the optimum value of the objective function

Exercise 6 : **Objective function** is a function which is to be maximised or minimised, subject to specific constraints on the variables

| SPM 2003 | SPM 2004 | SPM 2005 |
|---|---|---|
| By using graphs in exercise 5<br>What is the maximum amount of money that could remain from his allocation if yahya buys 10 kg of fish | By using graphs in exercise 5<br>Find (i) the maximum and minimum number of Mathematics participants when the number of Science participants is 10<br>(ii) the minimum cost to run the course | By using graphs in exercise 5<br>Find (i) the range of the number of participants for course Q if the number of participants for course P is 30<br>(ii) The maximum total fees per month that can be collected if the fees per month for courses P and Q are RM 50 and RM 60 respectively. |

Group Work ( 20 minutes )

*Use the graph paper to answer this question.*

1. A factory, produced 2 types of ping pong bats. A and B. In a day, the factory produces x units of type A and y units of type B. The factory uses two machines, P and W, to manufacture the bats. Machine P need 40 minutes and 20 minutes to produce type B bats respectively. Machine Q need 50 minutes and 20 minutes to produce a type A bat and a type B bat respectively. The production of bats by the factory in a day is subject to the following constraints :
(I) : The total time of usage of machine P has to be least than 400 minutes.
(II) : The total time of usage of machine Q cannot be more than 600 minutes
(III) : The number of type A bats produced cannot be more than 2 times the number of type B bats produced.

(a) Write three inequalities, apart from $x \geq 0$ and $y \geq 0$, which satisfy the above constraints

[3 marks]

(b) Using a scale of 2cm : 10 units on both axes, draw the graphs of the three inequalities. Hence, construct and shade the feasible region R that satisfies the given constraints.

[3 marks]

(c) Based on your graph, answer these following questions :
(i) : Find the maximum number of type A bats that can be produced by the factory in a day
(ii) : Find the minimum number of type B bats that can be produced by the factory in a day.
(iii) : The profits from the sales of a type A and a type B bat are RM4 and RM 6 respectively. Find the maximum profit that can be obtained in a day if the number of type B bats produced in a day is three times the number of type A bats produced.

[4 marks]

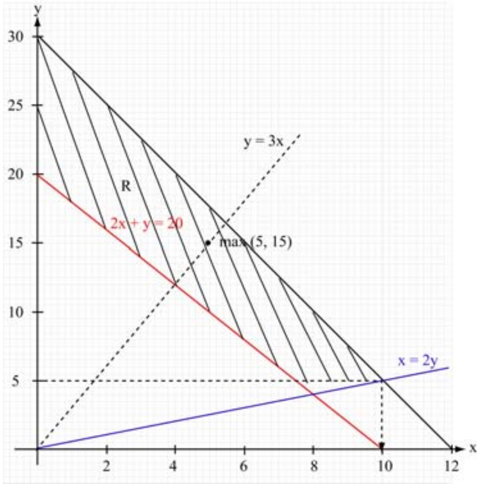
Answer

1) ( a ) $2x + y \geq 20$

$5x + 2y \leq 60$

$y \leq 2x$

( b )




( c ) ( i ) $x$ max = 10

( ii ) $y$ min = 4

( iii ) max. profit - 4 ( 5 ) + 6 ( 15 ) = 110

Quiz ( 15 minutes )

*Use the graph Provided by the invigilator to answer this question.*

A workshop produces two types of rack , P and Q. The production of each type of rack involves two processes, making and painting. Table 4 shows the time taken to make and paint a rack of type P and a rack of type Q

| Rack | Time taken ( minutes ) | |
|---|---|---|
| | Making | Painting |
| P | 60 | 30 |
| Q | 20 | 40 |

The workshop produces $x$ racks of type *P* and $y$ rack of type *Q* per day.
The production of racks per day is based on the following constraints:

I : The maximum total time for making both racks is 720 minutes

II : The total time for painting both rack is at least 360 minutes

III : The ratio of the number of rack of type P to the number of rack of type *Q* is at least 1: 3

a) Write three inequalities, other than $x \geq 0$ and $y \geq 0$ ,which satisfy all of the above constraints

[ 3 *marks* ]

b) Using a scale of 2 cm to 2 rack on both axes, construct and shade the region *R* which satisfies all of the above constraints.

[ 3 *marks* ]

c) By Using your graph find

(i) the minimum number of racks type *Q* if 7 racks of types *P* are produced per day,

(ii) the maximum total profit per day if the profit from one rack of type *P* is RM 24 and from one rack of type *Q* is RM 32

[ 4 *marks* ]