TEACHING AND LEARNING MODULE
ADDITIONAL MATHEMATICS FORM 5

PROBABILITY DISTRIBUTION

CHAPTER 8

NAME:………………………………………………….

FORM :………………………………………………..

Date received : ………………………………

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Formulae

a) $P(X = r) = {}^{n}C_{r}\, p^{r} q^{n-r}$ **,** $p + q = 1$

b) Mean $\mu = np$

c) $\sigma = \sqrt{npq}$

d) $z = \dfrac{x-\mu}{\sigma}$

LEARNING OUTCOMES

Students will be able to:

1. Understand and use the concept of binomial distribution.

1.1 List all possible values of a discrete variable..

1.2 Determine the probability of an event in a binomial distribution.

1.3 Plot binomial distribution graphs

1.4 Determine mean ,variance and standard deviation of a binomial distribution.

1.5 Solve problems involving binomial distributions.

BINOMIAL DISTRIBUTION

The binomial distribution is an example of a particular type of *discrete probability distribution.* It has relevance and importance in many real-life everyday applications. The binomial distribution may be referred to as a *Bernoulli distribution* , and the trials conducted are known as *Bernoulli trials* They were named in honour of the Swiss mathematician Jakob Bernoulli (1654–1705).

For a trial to be defined as a Bernoulli trial, each of the following characteristics must be satisfied.

1. n independent trials must be conducted.
2. Only two possible outcomes must exist for each trial — that is, success and failure.
3. The probability of success, p , is fixed for each trial.

If X represents a random variable which has a binomial distribution then it can be expressed as:

$X \sim Bi\ (n, p)$

Translated into words, $X \sim Bi\ (n, p)$ means that X follows a binomial distribution with parameters n (the number of trials) and p (the probability of success).

Consider the experiment where a fair dice is rolled four times. If X represents the number of times a 3 appears uppermost, then X is a binomial variable. Obtaining a 3 will represent a success and all other values will represent a failure. The die is rolled four times so the number of trials, n, equals 4 and the probability, p, of obtaining a 3 is equal to $\frac{1}{6}$. Using the shorthand notation, $X \sim Bi\ (n, p)$ becomes

$X \sim Bi\ (4, \frac{1}{p})$

We will now determine the probability of a 3 appearing uppermost 0, 1, 2, 3, and 4 times. Obtaining 3 is defined as a success and is denoted by ***S*** . All other numbers are defined as a failure and are denoted by ***F.*** The possible outcomes are listed in the table below.

Occurrence of 3	Possible outcomes	Probability	
0	*FFFF*	$1 \times \left(\frac{5}{6}\right)^4 = \frac{625}{1296}$	q^4
1	*SFFF FSFF FFSF FFFS*	$4 \times \left(\frac{5}{6}\right)^3 \left(\frac{1}{6}\right) = \frac{500}{1296}$	$4q^3 p$
2	*SSFF SFSF SFFS FSSF FSFS FFSS*	$6 \times \left(\frac{5}{6}\right)^2 \left(\frac{1}{6}\right)^2 = \frac{150}{1296}$	$6q^2 p^2$
3	*SSSF SSFS SFSS FSSS*	$4 \times \left(\frac{5}{6}\right)\left(\frac{1}{6}\right)^3 = \frac{20}{1296}$	$4qp^3$
4	*SSSS*	$1 \times \left(\frac{1}{6}\right)^4 = \frac{1}{1296}$	p^4

This procedure for determining the individual probabilities can become tedious, particularly once the number of trials increases. Hence if X is a binomial random variable, its probability is defined as follows.

P (X =x) = ${}^nC_x p^x q^{n-x}, x = 0,1,2,...., n$ **That is: x = the occurrence of the successful outcome.**

The formula may also be written as

P (X =x) = ${}^nC_x p^x (1-p)^{n-x}, x = 0,1,2,...., n$ **Here, the probability of failure, *q*, is replaced by 1 − *p*.**

Since this is a probability distribution, we would expect that the sum of the probabilities is 1. Therefore, for the above example: $\Pr(X = x) = \Pr(X = 0) + \Pr(X = 1) + \Pr(X = 2) + \Pr(X = 3) + \Pr(X = 4)$

$$= \frac{625}{1296} + \frac{500}{1296} + \frac{150}{1296} + \frac{20}{1296} + \frac{1}{1296} = 1$$

1. 0 Understand and use the concept of binomial distribution.
   1.1 List all possible values of a discrete variable..
   1.2 Determine the probability of an event in a binomial distribution.

A) Discrete random variables

1. A random variable is a quantity whose value cannot be predicted but is determined by the outcomes of an experiment
2. A random variable with a **countable** number of possible outcomes is call **discrete random variable** Example A fair dice is rolled three times and the number of times of getting the number "4" is recorded . Let *X* be the number of times of getting the number 4. The possible outcomes of *X* are *X* = { 0, 1,2,3 }. Thus, *X* is a discrete random variable.

Example 1

a) If *X* represents the number of married women in a group of five women, list all possible values of *X*	b) If *A* represents the number of boys of a family of four children, list all possible values of *A*	c) b) If *B* represents the number of rainy days in a particular of week *B* list all possible values of *B*

Exercise 1

a) If *X* represent the number of days that sally is late for school in a five-day week	b) A fair coin is tossed three times. If Y represents the number of times of getting tails in the three tosses, list all possible values of Y	c) Ali attempts to answer all the ten multiple-choice question. If X represent the number of correct answer obtained by Ali list all possible values of X

Probability Of An Event In A Binomial Distribution

Example 1

a) From past records, it is found that 8% of the light bulbs produced by a manufacturer are defective. A sample of 5 light bulbs is randomly selected from the production line. What is the exactly one of the light bulbs is defective? [answer 0.2866]	b) From previous experience, a marksman is found to have a 90% success rate of scoring a bulls eye. If the marksman fires 8 times, what is the probability that the marksman is successful on all 8 shots. { answer 0.4305]

Exercise 1

a) Based on past records, 30% of the eggs produced in a farm can be classified as grade A. A sample of 6 eggs is selected at random. Find the probability that only 4 eggs can be classified as grade A [answer 0.0595]	b) In a quick test, a candidate has to answer 10 multiple-choice questions where each question has 4 possible responses with only one correct answer. To Pass the test, one must obtain at least 5 correct answer. If the candidate decides to guess the answer for all question, what is the probability that the candidate that the candidate just passes the test .[answer 0.0584]

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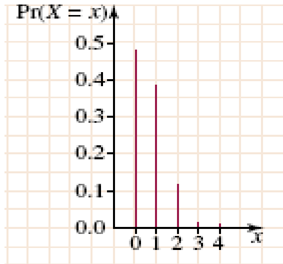
GRAPHS OF THE BINOMIAL DISTRIBUTION

We will now consider the graph of a binomial distribution. If we refer to the example of obtaining a 3 when rolling a die four times we note that $X \sim B\left(4, \frac{1}{6}\right)$. The probability distribution of the random variable, X, is given in the table and graph below.

x	$\Pr(X = x)$
0	0.4823
1	0.3858
2	0.1157
3	0.0154
4	0.0008



Example 2

a) A recent survey shows that 40% of the residents in town A have fixed deposit accounts. A sample of 3 resident is selected at random and the number of resident who have fixed deposit accounts, X is noted Determine the probability of all the events in the distribution and hence draw the graph of the distribution

Solution :

Example 2

a) A fair coin is tossed three times. If *X* represents the number of times heads obtained in the three tosses, determine the probability distribution of *X* . Hence, Plot the graph of the distribution of *X*

Solution

Homework Text book Page 199Exercise 8.1 No 9 – 11

Mean, Variance And Standard Deviation Of The Binomial Distribution

When working with the binomial probability distribution, (like other distributions) it is very useful to know the expected value (mean), variance and the standard deviation The random variable, X, is such that $X \sim B(8, 0.3)$ and has the following probability distribution.

x	0	1	2	3	4	5	6	7	8
Pr$(X = x)$	0.057 65	0.197 65	0.296 48	0.254 12	0.136 14	0.046 68	0.010 00	0.001 22	0.000 07

Mean :

$E(X) = \sum x \Pr(X = x)$. Hence, the expected value for the above table is:

$$\begin{aligned} E(X) &= \sum x \Pr(X = x) \\ &= 0 \times 0.057\,65 + 1 \times 0.197\,65 + 2 \times 0.296\,48 + 3 \times 0.254\,12 + 4 \times 0.136\,14 + \\ &\quad 5 \times 0.046\,68 + 6 \times 0.010\,00 + 7 \times 0.001\,22 + 8 \times 0.000\,07 \\ &= 0 + 0.197\,65 + 0.592\,96 + 0.762\,36 + 0.544\,56 + 0.233\,400 + 0.060\,00 + \\ &\quad 0.008\,54 + 0.000\,56 \\ &= 2.4 \end{aligned}$$

The variance was defined by the rule $\text{Var}(X) = E(X^2) - [E(X)]^2$. Hence, the variance for the above table is:

$$\begin{aligned} \text{Var}(X) &= E(X^2) - [E(X)]^2 \\ &= 0^2 \times 0.057\,65 + 1^2 \times 0.19765 + 2^2 \times 0.296\,48 + 3^2 \times 0.254\,12 + 4^2 \times 0.136\,14 + \\ &\quad 5^2 \times 0.046\,68 + 6^2 \times 0.010\,00 + 7^2 \times 0.001\,22 + 8^2 \times 0.000\,07 - (2.4)^2 \\ &= 0 + 0.197\,65 + 1.185\,92 + 2.287\,08 + 2.178\,24 + 1.167\,00 + 0.360\,00 + \\ &\quad 0.059\,78 + 0.004\,48 - (2.4)^2 \\ &= 7.440\,15 - (2.4)^2 \\ &= 1.68 \end{aligned}$$

The standard deviation was defined by the rule $SD(X) = \sqrt{\text{Var}(X)}$. Hence the standard deviation for the above table is:

$$SD(X) = \sqrt{\text{Var}(X)} = \sqrt{1.68} = 1.30$$

Since the above method for obtaining the expected value, variance and the standard deviation is tedious and time consuming, a quicker method has been developed to calculate these terms. It can be shown that

if $X \sim \text{Bi}(n, p)$ then:

$$E(X) = np$$
$$\text{Var}(X) = npq$$
$$SD(X) = \sqrt{npq}$$

To check that these agree with the previous example, we will substitute the values into the given rules. When $X \sim \text{Bi}(8, 0.3)$, we obtain the following.

The expected value:

$$\begin{aligned} E(X) &= np \\ &= 8 \times 0.3 \\ &= 2.4 \end{aligned}$$

The variance:

$$\begin{aligned} \text{Var}(X) &= npq \\ &= 8 \times 0.3 \times 0.7 \\ &= 1.68 \end{aligned}$$

The standard deviation:

$$\begin{aligned} SD(X) &= \sqrt{npq} \\ &= \sqrt{1.68} \\ &= 1.30 \end{aligned}$$

As can be seen, these values correspond to those obtained earlier. A great deal of time is saved using these rules and the margin for making mistakes is greatly reduced.

Hence, if X is a random variable and $X \sim \text{Bi}(n, p)$ then:

$$\mathbf{E(X) = np} \qquad \mathbf{Var(X) = npq} \qquad \mathbf{SD(X) = \sqrt{npq}}$$

Note: The distribution must be binomial for these rules to apply.

The random variable X follows a binomial distribution such that $X \sim \text{Bi}(40, 0.25)$. Determine the:

a expected value **b** variance and standard deviation.

THINK	WRITE
a 1 Write the rule for the expected value.	**a** $E(X) = np$
2 List the values for n and p.	$n = 40, p = 0.25$
3 Substitute the values into the rule.	$E(X) = 40 \times 0.25$
4 Evaluate.	$= 10$
b 1 Write the rule for the variance.	**b** $\text{Var}(X) = npq$
2 Write down the values for n, p and q.	$n = 10, p = 0.25, q = 0.75$
3 Substitute the values into the rule.	$\text{Var}(X) = 40 \times 0.25 \times 0.75$
4 Evaluate.	$= 7.5$
5 Write the rule for the standard deviation.	$\text{SD}(X) = \sqrt{npq}$
6 Substitute the value obtained for the variance and take the square root.	$= \sqrt{7.5}$
7 Evaluate.	$= 2.74$

A fair die is rolled 90 times. Find:

a the expected number of even numbers **b** the variance of even numbers

c the standard deviation of even numbers.

A binomial random variable has an expected value of 14.4 and a variance of 8.64. Find:

a the probability of success, p **b** the number of trials, n.

4 A fair coin is tossed 10 times. Find:

a the expected number of heads

b the variance for the number of heads

c the standard deviation for the number of heads.

5. Six out of every 10 cars manufactured are white. Twenty cars are randomly selected. Find:

a the expected number of white cars

b the variance for the number of white cars

c the standard deviation for the number of white cars.

A binomial random variable has a mean of 10 and a variance of 5. Find

(a) the probability of success, p

(b) the number of trials, n.

Homework Text book Page 199Exercise 8.1 No 9 – 11

The Normal Distributions

The normal distribution is an important tool when dealing with the probability distribution of a continuous random variable. The frequency curve of the normal distribution is characterised by the symmetrical bell shape called the

normal distribution curve or *normal curve* The normal curve fairly realistically models many observed frequency

distributions such as heights and weights of infants, Mathematical Methods examination results, the intelligence quotient of children in a particular age group, the lengths of battery lives, the diameters of steel cans, etc.

If X is a continuous random variable which follows a normal distribution with mean, μ and , and variance, σ^2 it is written as $X \sim N(\mu, \sigma^2)$. The standard normal distribution is written as $X \sim N(0, 1^2)$.

Z values are also known as standard score

Example 4. If *Z* is a standard normal variable, find the value of each of the following

[Jb a) 0.2119 b) 0.9834 c) 0.0968 d) 0.1673 e) 0.3239

a) P(Z $\geq$ 0.8)	b) P (Z $\leq$ 2.13	c) P (Z$\leq$ - 1.3)	d) P (0.8 $\leq$Z $\leq$1.7)	P (-0.24 $\leq$Z$\leq$0.61)

Exercise 4 **If *Z* is a standard normal variable, find the value of each of the following**

[Answer a) 0.2417 b) 0.8185 c) 0.6987 d) 0.025 e) 0.7704

a) P(0.5 < Z < 1.5)	b) P (-2 < Z < 1)	c) P (-0.93 $\leq$ Z $\leq$1.15	d) P (Z $\geq$1.96)	e) P (Z > - 0.74)

Example 5 : Z is a random variable having the standard normal distribution. Find the value of *a* such that

P(Z>a)= 0.3446 [Ans 0.4]	b) $P(Z < a) = 0.1841$ [Ans –0.9]	c) $P(Z \leq a) = 0.6406$ [Ans 0.36]	d) $P(Z \geq a) = 0.8508$ Ans– 1.04]

Example 5 : Z is a random variable having the standard normal distribution. Find the value of *b* such that

P(Z>b)= 0.2206 [Ans 0.77]	b) $P(Z < b) = 0.281$ [Ans - 0.58]	c) $P(Z \leq b) = 0.8686$ [jAns 1.12]	d) $P(Z \geq b) = 0.8238$ [Ans – 0.93]
P(Z>b = 0.0418 [Ans 1.73]	b) $P(Z < b) = 0.1093$ [jAns –1.23]	c) $P(Z \leq b) = 0.6736$ [Ans 0.45]	d) $P(Z \geq b) = 0.6141$ [Ans – 0.29]

Homework Text book Page 208 Exercise 8.2 No 2-3

Standardising A Normal Distribution

Each normal distribution has its own values of μ and σ .To simplify the process of determining the probability, a random variable *X* (x- value) in any normal distribution can be converted into a standardised variable,
Z (*z* – value). Subsequently, the probability can be determined using the standard normal distribution.

An *x*- value can be converted to a z – value by using the formula $Z = \dfrac{X - \mu}{\sigma}$ where *X* is the random variable (*x* – value) of the normal distribution with mean, μ and standard deviation, σ .

Example 6 a [answer 1) a) 3 b) – 0.875 b) a) x = 14.25 b) x = 10.944

1) *X* has a normal distribution with a mean of 40 and a standard deviation of 8. Convert the following *x*-values to *z*- values a) x = 64 b) x = 33	2) Determine the x – value for each of the following z – values for a normal distribution with μ = 16 and $\sigma = 3$ a) z = 2.125 b) z = 0.472

Exercise6 a [answer 1) a) – 0.5 b) 4 2) a) 19.5 b) 15

1) *X* has a normal distribution with a mean of 16 and a standard deviation of 3 . Convert the following *x*-values to *z*- values a) *x* = 14.5 b) *x* = 28	2) Determine the x – value for each of the following z – values for a normal distribution with μ = 30 and 5 a) *z* = - 2.15 b) z = - 3

Represent Probability Of An Event Using Set Notation and Determine the probability of an event

Example 7	b) Exercise 7
a) The lengths of pencils produced by a certain factory are normally distributed with a mean of 19 cm and a standard deviation of 0.05 cm . A pencil is randomly selected from the factory. Write, in set notation, and find the probability that the length of the selected pencil is a) more than 19 cm b) less than 18.7 c) between 18.9 cm and 19.4 cm	The mass of a loaf bread baked by a bakery is normally distributed with a mean of 420g and a standard deviation of 12g. A loaf of bread is chosen at random from the bakery . Find the probability that the mass of the loaf of bread chosen is a) at most 414 g (Ans 0.3085) b) more than 405 g (Ans 0.8994) c) between 409g and 430g 9 (Ans 0.6179)

Problems Involving Normal Distributions.

1 The mass of water-melons produced from an orchard follows a normal distribution with the mean of 3.2kg and a standard deviation of 0.5 kg. Find (i) the probability that a water-melon chosen randomly from the orchard has a mass of not more than 4.0 kg ii) the value of m if 60 % of the water-melon from the orchard have a mass more than m kg. (6 m) **Answer** a) i) 0.2936 ii) 0.79691 b) i) 0.9452 ii) 3.0735	**2** The mass of babies born in a maternity hospital follows a normal distribution with a mean of 3.1 kg and a standard deviation of 0.8 kg. Find a) the probability that a baby chosen at random from the hospital has a mass of not more than 3.8k (ans 0.8092) b) the value of a if 75% of the babies born in the hospital have a mass of more than a kg (ans 2.561)

STANDARD NORMAL DISTRIBUTION: Table Values Represent AREA to the LEFT of the Z score.

Kebarangkalian Hujung Atas *Q*(z) Bagi Taburan Normal *N*(0,1)

z	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
											SUBTRACT								
0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641	4	8	12	16	20	24	28	32	36
0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247	4	8	12	16	20	24	28	32	36
0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859	4	8	12	15	19	23	27	31	35
0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483	4	7	11	15	19	22	26	30	34
0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121	4	7	11	14	18	22	25	29	32
0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776	3	7	10	14	17	20	24	27	31
0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451	3	7	10	13	16	19	23	26	29
0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148	3	6	9	12	15	18	21	24	27
0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867	3	5	8	11	14	16	19	22	25
0.9	.1841	.1914	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611	3	5	8	10	13	15	18	20	23
1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379	2	5	7	9	12	14	16	19	21
1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170	2	4	6	8	10	12	14	16	18
1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985	2	4	6	7	9	11	13	15	17
1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823	2	3	5	6	8	10	11	13	14
1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681	1	3	4	6	7	8	10	11	13
1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559	1	2	4	5	6	7	8	10	11
1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455	1	2	3	4	5	6	7	8	9
1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367	1	2	3	4	4	5	6	7	8
1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294	1	1	2	3	4	4	5	6	6
1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233	1	1	2	2	3	4	4	5	5
2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183	0	1	1	2	2	3	3	4	4
2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143	0	1	1	2	2	2	3	3	4
2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110	0	1	1	1	2	2	2	3	3
2.3	.0107	.0104	.0102								0	1	1	1	1	2	2	2	2
				$.0^{2}990$	$.0^{2}964$	$.0^{2}939$	$.0^{2}914$				3	5	8	10	13	15	18	20	23
								$.0^{2}889$	$.0^{2}866$	$.0^{2}842$	2	5	7	9	12	14	16	18	21
2.4	$.0^{2}820$	$.0^{2}798$	$.0^{2}776$	$.0^{2}755$	$.0^{2}734$						2	4	6	8	11	13	15	17	19
						$.0^{2}714$	$.0^{2}695$	$.0^{2}676$	$.0^{2}657$	$.0^{2}639$	2	4	6	7	9	11	13	15	17
2.5	$.0^{2}621$	$.0^{2}604$	$.0^{2}587$	$.0^{2}570$	$.0^{2}554$	$.0^{2}539$	$.0^{2}523$	$.0^{2}508$	$.0^{2}494$	$.0^{2}480$	2	3	5	6	8	9	11	12	14
2.6	$.0^{2}466$	$.0^{2}453$	$.0^{2}440$	.0'427	$.0^{2}415$	$.0^{2}402$	$.0^{2}391$	$.0^{2}379$	$.0^{2}368$	$.0^{2}357$	1	2	3	5	6	7	8	9	10
2.7	$.0^{2}347$	$.0^{2}336$	$.0^{2}326$	$.0^{2}317$	$.0^{2}307$	$.0^{2}298$	$.0^{2}289$	$.0^{2}280$	$.0^{2}272$	$.0^{2}264$	1	2	3	4	5	6	7	8	9
2.8	$.0^{2}256$	$.0^{2}248$	$.0^{2}240$	$.0^{2}233$	$.0^{2}226$	$.0^{2}219$	$.0^{2}212$	$.0^{2}205$	$.0^{2}199$	$.0^{2}193$	1	1	2	3	4	4	5	6	6
2.9	$.0^{2}187$	$.0^{2}181$	$.0^{2}175$	$.0^{2}169$	$.0^{2}164$	$.0^{2}159$	$.0^{2}154$	$.0^{2}149$	$.0^{2}144$	$.0^{2}139$	0	1	1	2	2	3	3	4	4
3.0	$.0^{2}135$	$.0^{2}131$	$.0^{2}126$	$.0^{2}122$	$.0^{2}118$	$.0^{2}114$	$.0^{2}111$	$.0^{2}107$	$.0^{2}104$	$.0^{2}100$	0	1	1	2	2	2	3	3	4
3.1	$.0^{3}968$	$.0^{3}935$	$.0^{3}904$								3	6	9	13	16	19	22	25	28
				$.0^{3}874$	$.0^{3}845$	$.0^{3}816$	$.0^{3}789$				3	6	8	11	14	17	20	22	25
								$.0^{3}762$	$.0^{3}736$	$.0^{3}711$	2	5	7	10	12	15	17	20	22
3.2	$.0^{3}687$	$.0^{3}664$	$.0^{3}641$	$.0^{3}619$	$.0^{3}598$						2	4	7	9	11	13	15	18	20
						$.0^{3}577$	$.0^{3}557$	$.0^{3}538$	$.0^{3}519$	$.0^{3}501$	2	4	6	8	9	11	13	15	17
3.3	$.0^{3}483$	$.0^{3}466$	$.0^{3}450$	$.0^{3}434$	$.0^{3}419$						2	3	5	6	8	10	11	13	14
						$.0^{3}404$	$.0^{3}390$	$.0^{3}376$	$.0^{3}362$	$.0^{3}349$	1	3	4	5	7	8	9	10	12
3.4	$.0^{3}337$	$.0^{3}325$	$.0^{3}313$	$.0^{3}302$	$.0^{3}291$	$.0^{3}280$	$.0^{3}270$	$.0^{3}260$	$.0^{3}251$	$.0^{3}242$	1	2	3	4	5	6	7	8	9
3.5	$.0^{3}233$	$.0^{3}224$	$.0^{3}216$	$.0^{3}208$	$.0^{3}200$	$.0^{3}193$	$.0^{3}185$	$.0^{3}178$	$.0^{3}172$	$.0^{3}165$	1	1	2	3	4	4	5	6	7
3.6	$.0^{3}159$	$.0^{3}153$	$.0^{3}147$	$.0^{3}142$	$.0^{3}136$	$.0^{3}131$	$.0^{3}126$	$.0^{3}121$	$.0^{3}117$	$.0^{3}112$	0	1	1	2	2	3	3	4	5
3.7	$.0^{3}108$	$.0^{3}104$	$.0^{3}100$	$.0^{4}96$	$.0^{4}92$	$.0^{4}88$	$.0^{4}85$	$.0^{4}82$	$.0^{4}78$	$.0^{4}75$									
3.8	$.0^{4}72$	$.0^{4}69$	$.0^{4}67$	$.0^{4}64$	$.0^{4}62$	$.0^{4}59$	$.0^{4}57$	$.0^{4}54$	$.0^{4}52$	$.0^{4}50$									
3.9	$.0^{4}48$	$.0^{4}46$	$.0^{4}44$	$.0^{4}42$	$.0^{4}41$	$.0^{4}39$	$.0^{4}37$	$.0^{4}36$	$.0^{4}34$	$.0^{4}33$									