M o d u l e

M o d u l e

TEACHING & LEARNING

ADDITIONAL MATHEMATICS FORM 5

Progressions

CHAPTER 1

NAME:.....

FORM :.....

Date received : .....

Date completed .....

Marks of the Topical Test : .....

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Formulae/ Glossary

Progression ; A sequence of terms with certain characteristics in which there is a constant relaion between two successive terms

Series : A summation of the terms in a sequence.

Examples:

S = 1 + 2 + 3 + 4 + 5 = 15.

S = -2 + 4 - 6 + 8 - 10 + 12 - 14 = -8

Arithmatics Progression

a) $T_n = a + (n-1)d$

b) $S_n = \frac{n}{2}[2a + (n-1)d]$

Geometric Progression

a) $T_n = ar^{n-1}$

b) $S_n = \frac{a(r^n - 1)}{r - 1} = \frac{a(1 - r^n)}{1 - r}$, $(r \neq 1)$

c) $S_\infty = \frac{a}{1 - r}$, $|r| < 1$

FROM CS

Students will be able to:

1. Understand and use the concept of arithmetic progression.

1.1 Identify characteristics of arithmetic progressions.

1.2 Determine whether a given sequence is an arithmetic progression.

1.3 Determine by using formula:

a) specific terms in arithmetic progressions;

b) the number of terms in arithmetic progressions.

1.4 Find:

a) the sum of the first n terms of arithmetic progressions.

b) the sum of a specific number of consecutive terms of arithmetic progressions.

c) the value of n, given the sum of the first n terms of arithmetic progressions.

1.5 Solve problems involving arithmetic progressions.

1.1 Identify characteristics of arithmetic progressions.

Arithmetic Progression is a sequence of numbers where the difference between successive (consecutive) term is a constant.

Example 1 . Determine whether each of the following number sequences is an arithmetic progression or not

a) 4, 7, 10, 13, 16, ……..	b) 3, -1, -5, -9, …….	b) 1, 4, 9, 16, …….	3a, 3a-b, 3a-2b, 3a-3b, ...

Exercise 1 . Determine whether each of the following number sequences is an arithmetic progression or not

a) 1 , 3 , 6, 10,15 ……..	b) 2.0, 2.2, 2.4,2.6, 2.8 …	b) 9, 6, 3, 0, …….	$\frac{4}{3}, \frac{11}{6}, \frac{7}{3}$ ……

1.2 Determine whether a given sequence is an arithmetic Progression

If a sequence is an arithmetic progressions (AP) then the difference between the successive terms in the sequence is a constant. This constant is called the **common difference** and denoted by the letter ,***d***

Example 2 : Determine which of the following sequences is an arithmetic progressions

a) 2,6,10,14,……	b) 1,3 ,27 ,……..	d) $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}$,.....

Exercise 2 Determine which of the following sequences is an arithmetic progressions

a) *1,3,9,27 ,……*	b) 2, 6, 10, 14 ,……..	(c) 11,6,1,-4,…..

Homework Text Book Exercise 1.1 page 10 No 1 and 2

1.3 Determine by using formula: a) specific terms b) the number of terms in arithmetic progressions

An Arithmetic Progression (AP) can be represented as *a, a + d, a + 2d, a + 3d,a + 4d,……..*
where ***a*** is the first term and ***d*** *is* the common difference. You see that
the 2 nd term is $a + d$
the 3rd term is $a + 2d$ i.e. $a + (3 – 1)d$
the 4th term is $a + 3d$ ie. $a + (4 - 1)d$ and
the 5th term is $a + 4d$ i.e. $a + (5 -1)d$
The 20th term will be $a + (20 – 1)d = a + 1\ 9d$.
So, in general, the n^{th} term , T_n is represented by **$T_n = a + (n-1)\ d$** ***n*** **is the number of the terms**

Example 3 Find (a) the common difference (b) the 10th term and (c) the nth term for the following Arithmetic Progressions:

(i) 2, 5, 8, 11,…. [Ans 3,29,3n-1]	ii) 4, $6\frac{1}{2}$, 9 ,… [Ans 2 ½ ,26 ½ ½ (5n+3)]	(iii) 7, 5 ·5 , 4 [Ans –1.5,-6.58.5-1.5n]

Exercise 3 Find (a) the common difference (b) the 6^{th} term and (c) the n^{th} term for the following Arithmetic Progressions:

(i) $1\frac{1}{4}, 2\frac{1}{2}, 3\frac{3}{4}, \ldots$	ii) $\frac{4}{3}, \frac{11}{6}, \frac{7}{3}, \ldots$	(iii) 10, 4,-2,……

Example 4 Find the number of terms in the-following arithmetic progressions :

(a) 12, 15,18……36 [Ans 9]	(b) 10,4,-2…….. – 38 [Ans 9]	c) $m - 2n$, m , $m+2n$,.. , $m + 18n$ [Ans 11]

Exercise 4 Find the number of terms in the-following arithmetic progressions :

(a) 5, 10, 15,……. 65 [Ans13]	(b) 4,1, -2,……..,-26 [Ans 11]	d) a+2x, a, a-2x,…a - 38x. [Ans 21]]

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Example 5

a) The 7 th term and the 12 th term of an arithmetic progression are 27 and 47 respectively . Find the 30 th term [Answer 119]	b) The 13 th term of an AP is 27. Given that the 7 th term is 3 times than the second terms, find the first term and the common difference. [Answer a = 3 , d = 2]

Exercise 5

a) The 6 th term and the 16 th term of an arithmetic progression are 25 and 85 respectively . Find the 25 th term [Answer 139]	b) If 3m – 1 , 6m and 19 are three consecutive terms in AP find the value of m [m = 2]

Homework Text Book Exercise 1.1 page 10 No 9 – 13

1.4 Find:

a) the sum of the first n terms of arithmetic progressions.
b) the sum of a specific number of consecutive terms of arithmetic progressions.
c) the value of n, given the sum of the first n terms of arithmetic progressions

Let the sum of the first *n* terms is S_n

Then $S_n = a + (a+d) + (a+2d) + (a+3d) + \ldots + (a+(n-1)d)$.

By reversing the terms of the series on the right-hand side, we have :

$S_n = (a + (n-1)d) + (a + (n-2)d) + (a + n-3d) + \ldots + a$

Adding,

$2S_n = (2a + (n-1)d) + (2a + (n-1)d) + \ldots\ldots$ up to *n* terms

$= n(2a + (n-1)d)$

$\Rightarrow \quad \mathbf{S_n} = \frac{n}{2}[2a+(n+1)d]$

For an A.P. of *n* terms

a, a +*d,* a +2d …… *l* where the last term l = a + *(n-1)d,* the sum of *n* terms as obtained from above is

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$= \frac{n}{2}[a + a + (n-1)d]$$

$$= \frac{n}{2}[a + l]$$

i.e. if the sum of *n* terms of an A.P. with the first term a and the last term l is S_n,then

$S_n = \frac{n}{2}[a+l] \Rightarrow S_n = n \times \frac{a+l}{2}$

= *n* x average of the first and the last term

Example 6 Find the sum of each of the following series/or AP

a) 50,42,34,….-22 [Ans 140]	(b) 4 + 7 + 10 + ... to 15 terms [Ans 375]	(d) -10 -7- 4 - ..to 14 terms [Ans 133]	(e) 4a + (3a + b) + (2a + 2b) +…to 12 terms [Ans 66b – 18a]

Exercise 6 Find the sum of each of the following AP.

a) 1,2,3,4,…….100 [Ans 5050]	b) 101,113,125 ….to 20 terms [ans 4300]	(a) 11+13+15 +.... to l2 terms [Ans 264]

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Example 7

a) The n th term for an AP is given by $T_n = 19 - 7n$ Find the sum of the first 15 terms. [Jb –555]	b) The first term for an AP is 13 and the sum of the first 25 terms is 875. Find i) the common difference [Ans 11/6] ii) the 21st term [Ans 49 2/3] iii) the sum of the first 12 terms [Ans 277]

Exercise 7

a) find the least number of terms of the AP 4 , 7,10…. that must be taken in order that the sum exceeds 125 [Ans 9]	b) In an AP the sum of the first 10 terms is 255 and the sum of the next 5 terms is 315 . Find the 7 th terms [Ans 33]

Homework Text Book Exercise 1.1 page 11 No 16 - 21

1.5 Solve problems involving arithmetic progressions

Example 8

a) Find the sum of all integers laying between 100 and 500 that can divisible by 17 [Ans 7140]	b) Ali pays back her loan monthly installments. He pays RM 20 in the first month . For each of the following months he pays RM 4 more than the previous month If Ali,s loan is RM 3572. find a) the numbers of months needed for Ali to clear his loan [Ans 38 month] b) the amount of installment in the final month.[ans RM 168]

Exercise 8

a). How many terms of the arithmetic progression 12 + 16 + 20 +…. must be taken for the sum to be equal to 672? (Camb.) [Ans 16]	b) The sum to 14 terms of the progression 1, 1· 4, 1·8, . … is equal to the sum of *n* terms of the progression 5·6, 5·8, 6·0,… Find the value of *n* [Ans 8]

Homework Text Book Exercise 1.1 page 12 No 22,23 24.

FROM CS

Students will be able to: 2. Understand and use the concept of geometric progression 2.1 Identify characteristics of geometric progressions. 2.2 Determine whether a given sequence is a geometric progression. 2.3 Determine by using formula: a) specific terms in geometric progression, b) the number of terms in geometric progressions.	2.4 Find: a) the sum of the first n terms of geometric progressions; b) the sum of a specific number of consecutive terms of geometric progressions. c) the value of n, given the sum of the first n terms of geometric progressions 2.5 Find: a) the sum to infinity of geometric progressions b) the first term or common ratio, given the sum to infinity of geometric progressions. 2.6 Solve problems involving geometric progressions.

2.1 Identify characteristics of geometric progressions.
2.2 Determine whether a given sequence is a geometric progression.

A Geometric *progression (G. P.)* is a sequence of terms in which each term is obtained from the preceding one by multiplying it by a constant. This constant is called the *common ratio. r .*

Example 9 : Determine which of the following sequences are geometric progressions: G.P

a) 1, 3, 9,27,……..	b) 1, 4,9,16,……..	c) $1,\frac{1}{2},\frac{1}{4},\frac{1}{16}$

Exercise 9

a) 1,2, 4,-8, 16,32	b) $\frac{1}{3},\frac{1}{12},\frac{1}{48},\frac{1}{192},$.....	c) 4, -12, 36,-108,…….

Homework Text Book Exercise 1.2 page 22 No 1 and 2

2.3 Determine by using formula:
a) specific terms in geometric progression b) the number of terms in geometric progressions.

The general term of a G.P.

A geometric progression can be represented as

$a, ar, ar^2, ar^3, ar^4, \ldots\ldots\ldots\ldots$ where a is the first *term* and r the common ratio of the G.P.

We see that

the 3rd term is ar^2 i.e. ar^{3-1}

the 4th term is ar^3 i.e. ar^{4-1}

and the 5th term *is* ar^4 *i.e.* ar^{4-1}

The 10th term will be $ar^{10-1} = ar^9$.

So, in general, the n^{th} term , T_n is represented by

$$T_n = ar^{n-1}$$

Example 10 : Find (a) the common ratio *(b)* the 9th *term (c)* the nth term for each of the following G.P:

(i) $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots\ldots$ [Ans ½ , ½ ^{n} , ½ $^{n-1}$]	(ii) 1,-1,1,-1,……. [Ans -1,1,$(-1)^{n-1}$]	(iii) 3,-6,12,-24,….. [ans $-2, 3(-2)^8, 3(-2)^{n-1}$]

Exercise 10 : Find (a) the common ratio *(b)* the 6th *term (c)* the nth term for each of the following G.P:

(a) 1 , 3,9, 27 …… [Ans 3,243, 3 $^{n-1}$]	b) 12, -4, $\frac{4}{3}$, $-\frac{4}{9}$, ……. [Ans – 1/3 , - 12/234 , $12(-1/3)^{n-1}$]	(iii) $\frac{1}{p}, 1, p, p^2, \ldots\ldots$ [Ans p ,p^4, p^{n-2}]

Example 10 :

Determine the number of terms in the following G.P 6,12,24,……..1536. [Ans 9]	b) How many terms is there in the following G.P 3 , -6 , 12 ,-24 ….768. [Ans9]	c) The 1 st term of a G.P is $\frac{5}{11}$ and the 6th term is $110\frac{5}{11}$. Find the common ratio and the n^{th} term [Ans 3, $\frac{5}{11}(3)^{n-1}$]

Exercise 10 :

Determine the number of terms in the following G.P 2,10,50,……, 31 250 . [Ans 7]	b) How many terms is there in the following G.P $\frac{1}{4},\frac{1}{12},\frac{1}{36},.....,\frac{1}{8748}$. [Ans8]	a) How many terms are there in the G:P. 3,6, 12 …., 3072? [Ans 11]

Homework Text Book Exercise 1.2 page 22 No 3,4,5

Example 11

a) In a G.P the first terms and the third terms exceeds the seconds terms by 12 and 24 respectively . Find the first term and the common ratio [ans $a = 4$, $r = -2$]	b)Find the first term of the geometric progression 144, 108, 81,……. that exceeds 10 [ans $n = 10$]

Exercise 11

a) If 5, x, y, z, 405 are first five terms of a G.P, find the value of x, y and z [Ans 15,45,135]	b) If x-1, x + 1, 2x + 2 are three consecutive terms of a G.P Find the value of x and $x \neq 0$. [Ans 3]

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2.4 Find: a) the sum of the first n terms of G.P b) the sum of a specific number of consecutive terms of G.P
c) the value of n, given the sum of the first n terms of G.P

Consider the GP of n terms : a , ar , ar^2 , ar^3 ,……ar^{n-1}

Let S_n be the sum of the first n terms of a G.P.

Hence $S_n = a + ar + ar^2 + ... + ar^{n-1}$ …….. (1)

Multiply throughout by r :

$rs_n = ar + ar^2 + \quad + ... + ar^{n-1} + ar^n$ ……(2)

(2) -(1) : $r\,Sn - Sn = ar^n - a$

$\Rightarrow (r-1)\,S_n = a(r^n - 1)$

$$S_n = \frac{a(r^n - 1)}{r - 1}, \quad r \neq 1$$

If we subtract (1) – (2) we get $S_n = \frac{a(1-r^n)}{1-r}$, $r \neq 1$

This formula is usually used if r is numerically less than 1, i.e $|r| < 1$.

Example 12

a) Find the sum of the first eight terms of the G.P 4,8,16,……. [Ans 1020]	b) Find the sum of the G.P 5,-10,20,……,320 [Ans 215]

Exercise 12

a) In the G.P 64 , -16 , 4 , -1 +……, Find the sum to 10 terms [Ans51]	b) Find the sum of the G.P 48,12,3,….., $\frac{3}{64}$ [Ans $63\frac{63}{64}$]

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Example 13

a) The 2^{nd} term and the 5^{th} term of a G.P. are 6 and 48 respectively. Find the sum of the first 10 terms. [Ans 3069]	b) Find the least number of terms of the G.P. I + 3 + 9 + 27 +…….that must be taken in order that the sum exceeds $1{\cdot}4 \times 10^5$ [Ans 12]

Exercise 13

a) In the G.P the sum of second term and third term is -48, while the sum of third terms and the fourth terms is 16. Find the sum of the first 5 terms.[ans $162\frac{2}{3}$]	b) The first term of a certain geometric progression is $2\frac{1}{2}$, and the sixth term is $607\frac{1}{2}$. Find (i) the common ratio *(ii)* the sum , correct to 3 significant figures, of the first 10 terms. [Ans 3, 73800,]

Homework Text Book Exercise 1.2 page 23 No 16 - 19

2.5 Find:

a) the sum to infinity of geometric progressions b) the first term or common ratio, given the sum to infinity of geometric progressions.

Sum to infinity, S∞

From the formula for the sum of *n* terms of a G.P. with $|r|$ < I, we see that

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$= \frac{a}{1-r} - \frac{ar^n}{1-r}$$ if $|r| < 1$, then as *n* increases, r^n *decreases,* and as $n \to \infty$,

Hence the sum S_n approaches a finite value $\frac{a}{1-r}$ as $\frac{ar^n}{1-r} \to 0$ when $n \to 0$

The value that S_n approaches as $n \to \infty$ is known as the *sum to infinity, S∞*

Thus when $|r|$ < I , the sum to infinity of a G.P. is $S_\infty = \frac{a}{1-r}$

Example 14

a) The first three terms of a G.P. are x + 10, x -2 and x – 10 respectively. Calculate the value of x and hence find (i) the common ratio (ii) the sum to infinity	b) Find the sum to infinity for the G.P i) 16, 8,4,2 …… [Ans 32] ii) *27, -9,* 3,-1….. [Ans $20\frac{1}{4}$]	c) The second term and the sum to infinity of a GP are 2 and 8 respectively. Find the sum of the first 6 terms. [Ans 7 7/8]

Exercise 14 Find the sum to infinity for the G.P.

a) $\frac{1}{2}, \frac{1}{8}, \frac{1}{32}$ Ans (2/3)	b) 2 , 1.8, 1.62 ,……. [ans 20]	c) If the sum to infinity of a G.P is two times of the first terms .find the common ratio of a G.P [ans r = ½]

Example 15 Express the recurring decimal as a fraction in its lowest terms

a) 0.3333….. [Jb1/3]	b) $0.\dot{0}2\dot{7}$ [Jb $\frac{1}{37}$]

Exercise 15 Express the recurring decimal as a fraction in its lowest terms

a) 0.8888 [Jb 8/9]	b) $0.\dot{3}\dot{6}$ [Jb $\frac{4}{11}$]

Homework Text Book Exercise 1.2 page 23 No 20 – 25]

2.6 Solve problems involving geometric progressions

Example 15

a) How many terms of the geometric progression 48 +24 + 12 + ….. must be taken for the sum to be equal to $95\frac{1}{4}$ [Ans 7]	b).Find the sum of the first *n* terms of the geometric progression 5 + 15 + 45 + What is the least number of terms which will give a total of more than 10^8? [ans 16]

Exercise 15

a) The first term of a G. P. is 8 and the common ratio 2 What is the minimum number of terms that must be taken so that the difference between the sum of these terms and the sum to infinity is less than 0.01? [ans 11]	a) If $3\frac{5}{9}$ and $40\frac{1}{2}$ are the first and last terms of a geometric progression and there are 7 term altogether, find the second term. (Camb.) [5 1/3]

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SPM Questions

SPM 2003	SPM 2004
a). The first three terms of an arithmetic progression are k - 3, k + 3, 2k + 2. Find (a) the value of k, (b) the sum of the first 9 terms of the progression. [3 marks] b. In a geometric progression, the first term is 64 and the fourth term is 27. Calculate (a) the common ratio, (b) the sum to infinity of the geometric progression. [4 marks]	**a**) Given a Geometic Progression y , 2, $\frac{4}{y}$, p …express p in terms of y [2 marks] b) Given an arithmatic progression -7, -3,1 … state three consecutive terms in this progression which sum up to 75 [3 marks] c) The volume of water in a tank is 450 litres on the first day. Subsequently, 10 linters of water is added to tank everyday. Calculate the valume, in litres, of water in the tank at end of the 7th day [2 marks] d) Express the recurring decimal 0.969696…as a frection in its simplest form [4marks]
SPM 2005	**SPM 2006**
a) The first three terms of a sequence are 2, x, 8 Find the positive value of x so that the sequence is i) an arithmatic progression ii) a geometric progression [2 marks] b) The first three terms of an arithmatic progression are 5,9,13. Find i) the common difference of the progression ii) the sum of the first 20 terms after the 3rd terms [4 marks] c) The sum of the first n terms of the geometric progression 8,24,72,….is 8744. Find i) the common ratio of the progression ii) the value of n [4 marks]	a) The 9 th term of an arithmatic progression is 4 + 5p and the sum of the first four terms of the progression is 7p – 10, where p is constant. Given that the common difference of the progression is 5, find the value of p. [3 marks] b) The third term of a geometric progression is 16. The sum of the third term and the fourth term is 8 Find i) the first term and the common ratio of the progression ii) the sum ti infinity of the progression [4 marks]

Topic 1 Progressions Enrichment

1. Find
   a) the common difference ,
   b) the 10th term ,
   c) the *n* th term for the following arithmetic progressions

   i) 2 , 5, 8 , 11 , ……. *Ans. : a) 3 b) 29 c) 3n - 1*

   ii) 4 , $6\frac{1}{2}$, 9 , ……. *Ans : a)* $2\frac{1}{2}$ *b)* $26\frac{1}{2}$ *c)* $\frac{1}{2}(5n+3)$

2. Find the number of terms in the following arithmetic series :
   a) 12 , 15 , 18 , ……., 36 *Ans. : 9*
   b) a + 2x , a , a – 2x , …….. , a – 38x. *Ans. : 21*

3. Find the sum of each of the following series :
   a) 11 + 13 + 15 + …….. to 12 terms. *Ans. : 264*
   b) -10 -7 – 4 - ……..to 14 terms. *Ans. : 133*

4. The first three terms of an arithmetic progression are *h* – 3, *h* + 3, 2*h* + 2. Find
   (a) the value of *h* *Ans. : h = 7*
   (b) the sum of the first 9 terms of the progression *Ans. :* S_9 *= 252*

5. Find
   a) the common ratio
   b) the 9 th term
   c) the *n* th term for each of the following geometric progressions

   i) 1 , 3 , 9 , 27 , ……. *Ans. : a) 3 b)* 3^8 *c)* 3^{n-1}

   ii) 1 , $\frac{1}{2},\frac{1}{4},\frac{1}{8}$, ……. *Ans. : a)* $\frac{1}{2}$ *b)* $(\frac{1}{2})^8$ *c)* $(\frac{1}{2})^{n-1}$

6. How many terms at least of the arithmetic progression 1 , 4 , 7 , 10, …….. are needed to give a sum greater than 590 , starting from the first term of the arithmetic progression?
   Ans. : n = 21

7. How many terms are there in each of the following geometric progressions ?
   a) 3 , -6 , 12 , -24 , …….. , 768. *Ans. : 9*
   b) 2 , 10 , 50 , ……., 1250 . *Ans. : 7*

8. Given a geometric progression $-\frac{1}{3}$, 1 , -3 , 9 ……. , which term is equal to 729 ?
   Ans. : The 8th term

9. Find the sum of each of the following series :
   a) 2 – 4 + 8 – 16 + …….. to 10 terms *Ans. : - 682*
   b) $2a + 4a^2 + 8a^3 +$ …….. to 11 terms *Ans. :* $\frac{2a}{1-2a}[1-(2a)^{11}]$

10. The 2nd and the 5 th term of a geometric progression are 6 and 48 respectively . Find the geometric progression and the sum of the first 10 terms.
    Ans. : The G.P is 3 , 6 , 12 , 24 , ……..
    and S_{10} *= 3069*

11. Find the sum to infinity for the geometric progressions
    a) 16 , 8 , 4 , 2 ,…….. *Ans. : 32*
    b) 27 , -9 , 3 , -1 ,……. *Ans. :* $20\frac{1}{4}$

SUMMATIVE EVALUATION

1 A piece of wire of length p cm is cut into 30 parts such that their lengths form an arithmetic progression. Given that the length of the longest part is 99 cm and the sum of the lengths of the three shortest parts is 45 cm. Find
   (a) the length of the shortest part,
   (b) the value of p.
   Ans : (a) 12 cm
   (b) p =1665

2 The sum of the first n terms of a geometric progression is given by $S_n = -16[1 - (½)^n]$. Find

(a) the first term,

(b) the common ratio,

(c) the sum of the first n terms, where n is large enough such that $r^n \approx 0$.

Ans : ***(a)*** ***–8***

(b) ***1/2***

(c) ***–16***

3. In a geometric progression, the sum of the first five terms is – 31/8. Given that the common ratio is 1/2. Find

(a) the first term,

(b) the sum of all the terms from the fourth to the sixth term.

Ans : ***(a)*** ***–2***

(b) ***–7/16***

4 The price of a new car is RM 60,000. Its price decreases by 10% every year. Find

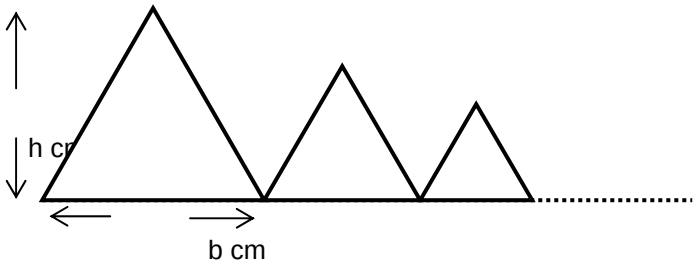
(a) the price of the car after 5 years.

(b) the percentage decrease in the price of the car in 5 years time.

Ans : ***(a)*** ***RM 35429.40***

(b) ***40.95%***

5. The following diagram shows the arrangement of the first three of an infinite series of similar triangles. The first triangle has a base of b cm and a height of h cm. The measurements of the base and height of each subsequent triangle are half of the measurements of its previous one.



(a) Show that the areas of the triangles form a geometric progression and state the common ratio.

(b) Given that b = 160 cm and h = 80 cm,

(i) determine which triangle has an area of 25 cm^2,

(ii) find the sum to infinity of the areas, in cm^2, of the triangles.

Ans : ***(a)*** ***1/4***

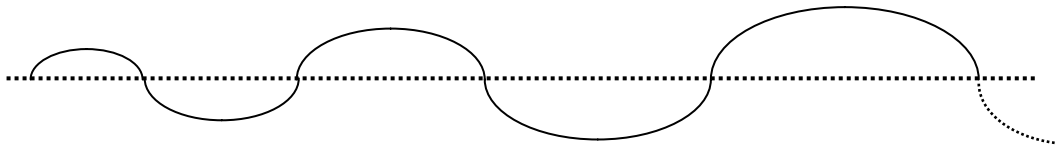
(b) ***(i)*** ***5th triangle***

(ii) 8533. 33

6 The perimeter of a hexagon is 36cm. The lengths of the sides of the hexagon are in arithmetic progression and the length of the longest side is five times the length of the shortest side. Find the length of the shortest side.

Ans : ***2 cm***

7



A piece of wire is bent to form arcs of semicircles, as shown in the above figure. The radius of the smallest semicircle is 3 cm and the radii of the following semicircles increase by 2 cm, in sequence.

(a) If the radius of the largest semicircle is 41cm, calculate the number of semicircles that are formed.

(b) Can the semicircle pattern in (a) be formed from a 360π cm long wire?

Ans : ***(a)*** ***20 semicircles***

(b) ***Only 18 semicircles can be formed. Hence the semicircle pattern in (a), which consists of 20 semicircles cannot be formed.***