

module module TEACHING & LEARNING

ADDITIONAL MATHEMATICS FORM 4

CIRCULAR MEASURES

CHAPTER 8

NAME:

FORM :

Date received :

Date completed

Marks of the Topical Test :

Additional Mathematics Department
Sek Men Sains Muzaffar Syah Melaka

For Internal Circulations Only

1 Arc length, $s = r\theta$

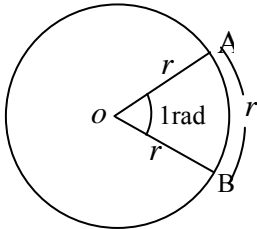
2 Area of sector, $L = \frac{1}{2}r^2\theta$

Students will be able to:
 Understand the concept of radian.
 1.1 Convert measurements in radians to degrees and vice versa.

1. Radian

Converting measurements in radians to degrees and vice versa.

a) Angles can be measure in degrees ($^{\circ}$) or radians (rad)



b) i) The angle subtended at the centre of a circle by an arc with the same length as the radius of the circle is define as 1 radian

ii) If the length of the arc AB = $2r$, then the angle subtended at the centre of the circle is 2 radians and so on

c) When the arc length = $2\pi r$ (circumference of the circle) then the subtended angle = 2π radian

Since the angle at the centre of a circle = 360° then $2\pi \text{ rad} = 360^{\circ}$

$$\pi \text{ rad} = 180^{\circ}$$

$$\text{Thus } 1 \text{ rad} = \frac{180^{\circ}}{\pi} = 57.3^{\circ}$$

$$1^{\circ} = \frac{\pi}{180} \text{ rad} = 0.01745 \text{ rad}$$

Example 1/ Exercise 1 Convert the following angles in radian to degree and minutes

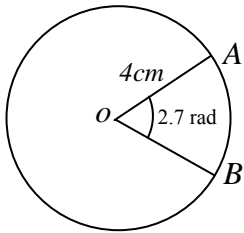
| | | | |
|--|--|--|--|
| a) $0.45\pi \text{ rad}$ Ans(81°) | e) $\frac{2}{3}\pi \text{ rad}$ Ans(120°) | c) $2.57\pi \text{ rad}$ Ans ($462^{\circ}36' @ 462.6^{\circ}$) | a) 3.75 rad Ans ($214^{\circ}50' @ 214.83^{\circ}$) |
|--|--|--|--|

Example 2 Convert the following angles in degree and minutes to radians

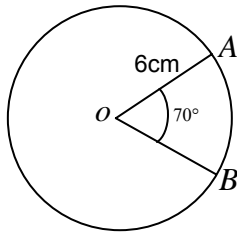
| | | | |
|---------------------------|---------------------------|-----------------------------|----------------------------|
| a) 75° (1.309) | b) 50.3° (0.878) | c) $180^{\circ}18'$ (3.147) | f) $256^{\circ}40'$ (4.48) |
|---------------------------|---------------------------|-----------------------------|----------------------------|

Example 3 Find the length of the arcs in each of the circles below

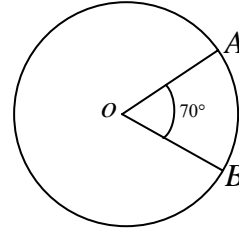
a) [Ans 10.8 cm]



b) ans [7.333]

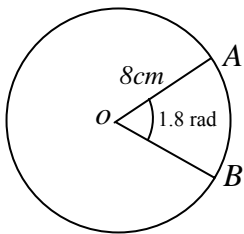


c) Given that the length of the major arc AB is 62 cm , find the length of the radius [$r = 12.25\text{cm}$]

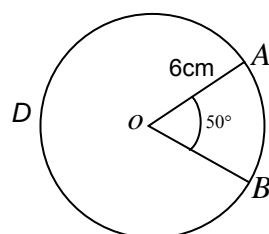


Exercise [Ans a) 14.4 cm b) 32.47 c) $= 114^{\circ}35'$]

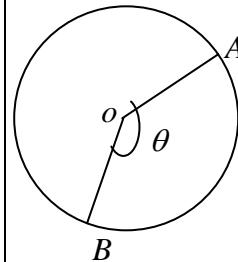
a) Find the length of AB



b) Find the length of ADB



c) Given that the length of arc AB is 15 cm and its radius is 7.5 cm find θ in degree and minutes

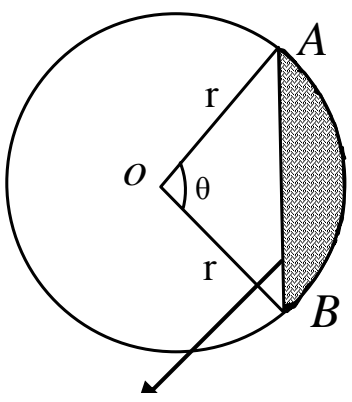


Homework Text Book exercise 8.2.1 page 183

Complete the table below by finding the values of θ , r or s .

| θ | r | s |
|--------------|-------|----------|
| 1.5 rad | 9 cm | |
| | 14 cm | 30 cm |
| 54° | | 2.828 cm |

2.2 Finding perimeter of segments of circles.



Line segment AB

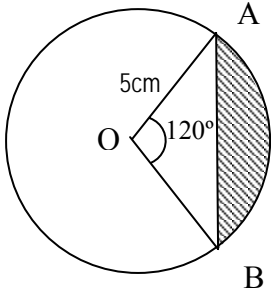
(a) Perimeter of the segment AB

$$= 2r \sin \frac{\theta}{2} + r\theta$$

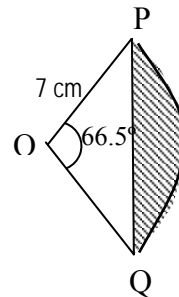
(b) For $\sin \frac{\theta}{2}$, θ is expressed in degrees

Example 3 [ans a) 19.1323 cm b) 20.96 cm

a) Find the perimeter of the shaded segments below

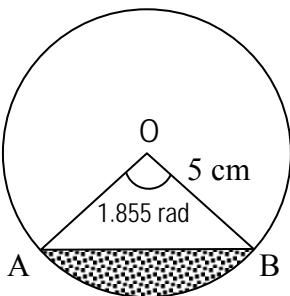


b) Find the perimeter of the shaded segment

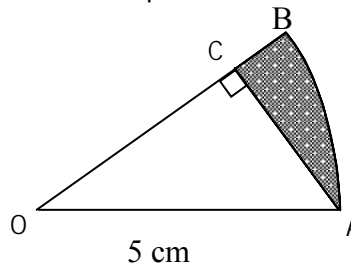


Exercise 3 [Ans 17.275 cm b) 7.218 cm]

a) Find the perimeter of the shaded segments below



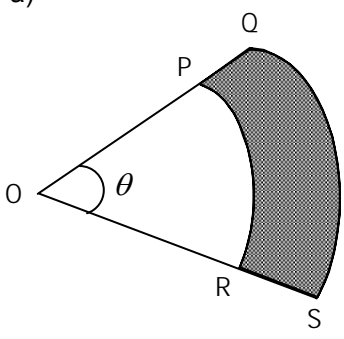
b) The diagram shows OAB is a sector of a circle with centre O and radius 5 cm. BC is 1 cm. Find the perimeter of the shaded segments below



2.3 Solving problems involving lengths of arc

Example 4 [Answer i) 0.75 rad ii) 14.5 cm] b) 13.09 cm)

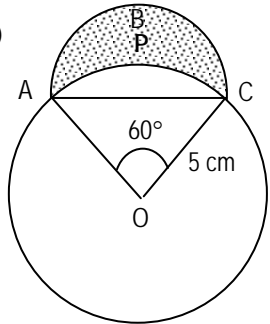
a)



The diagram shows two arc QS and PR , of two circles with centre O and with radii OS and OR . Given the ratio $OS : OR = 4 : 3$ and $RS = 2$ cm and $QS = 6$ cm find

- θ in rad
- perimeter of the shaded region

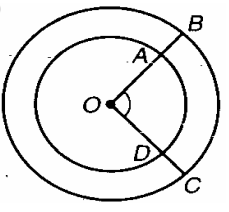
b)



The Diagram shows, O is a centre of a circle APC and ABC is a semicircle where AC is a diameter. Find the perimeter of the shaded region.

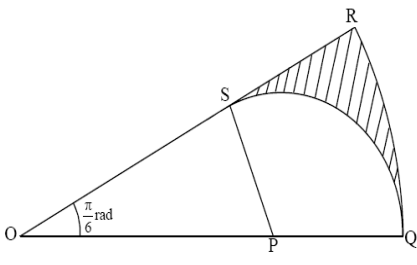
Exercise 4 [Answer a) $60^\circ 9'$ b) 8.4 cm^2

a)



The diagram shows two circle with centre O . Given that $OD = 8$ cm, $DC = 2$ cm and the length of arc $BC = 10.5$ cm. Find

- the angle of AOD in degree and minutes
- the length of arc AD

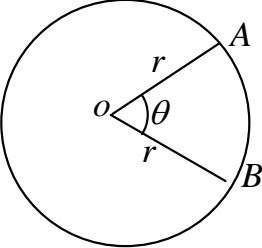
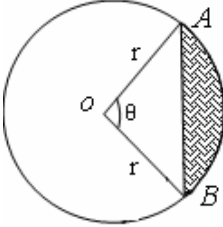


The diagrams shows OQR and PQS are sectors of circles with centers O and P respectively. PS is perpendicular to OR and the length of the circular arc QR is 8π . Calculate

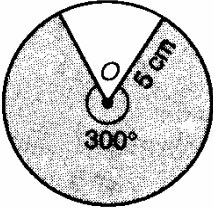
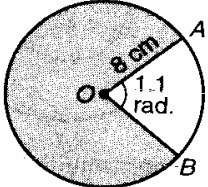
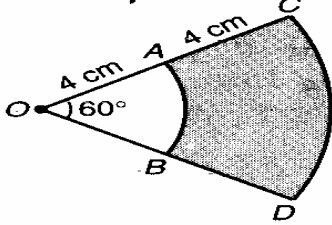
- the length of PS ,
- the perimeter of the shaded region,

Students will be able to:

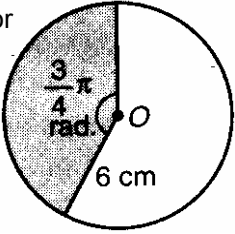
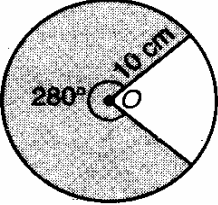
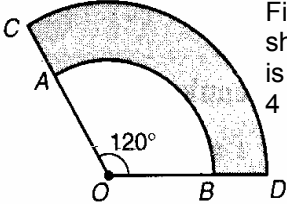
3. Understand and use the concept of area of sector of a circle to solve problems.
 - 3.1 Determine the: a) area of sector; b) radius; and c) angle subtended at the centre of a circle based on given information.
 - 3.2 Find the area of segments of circles.
 - 3.2 Solve problems involving areas of sectors.

| | | | |
|---|--|---|--|
|  | <p>Area of sector AOB is given by the formula, $A = \frac{1}{2} r^2 \theta$ where θ is an angle in radians</p> |  | <p>Area of segment AB is given by the formula, $A = \frac{1}{2} r^2 \theta - \frac{1}{2} bh$ where b = base h = heights of triangle AOB θ is an angle in radians</p> |
|---|--|---|--|

Example 5 Jb a) 65.46 cm^2 b) 165.888 cm^2 c) $8 \pi \text{ cm}^2$

| | | |
|--|--|--|
|  | <p>b) </p> <p>Find the area of the shaded sector</p> |  |
| <p>Find the area of the shaded sector</p> | | <p>Find the area of the shaded region</p> |

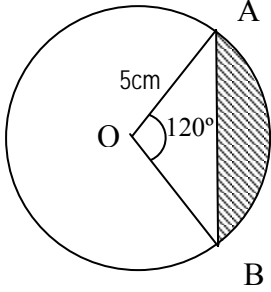
Exercise 5 [Ans 42.4 cm^2 b) 244.38 cm^2 c) $26 \frac{2}{3} \pi \text{ cm}^2$]

| | | |
|--|--|---|
| <p>a) Find the area of the shaded sector</p>  | <p>b) Find the area of the shaded sector</p>  | <p>Find the area of the shaded region if BO is 8 cm and BD is 4 cm.</p>  |
|--|--|---|

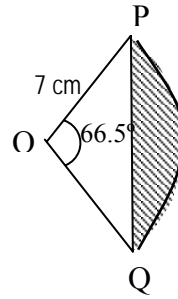
2.2 Finding the area of areas of circles.

Example 6 [ans a) 15.3547 cm^2 b) (5.96 cm^2)

b) Find the area of the shaded segments below

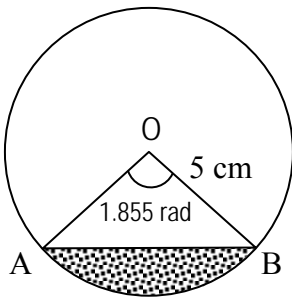


b) Find the area of the shaded segment

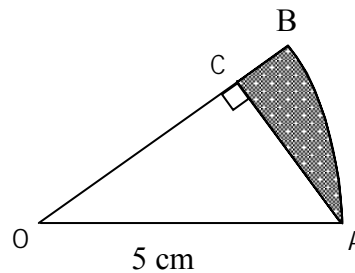


Exercise 6 [Ans (11.19 cm^2) b)]

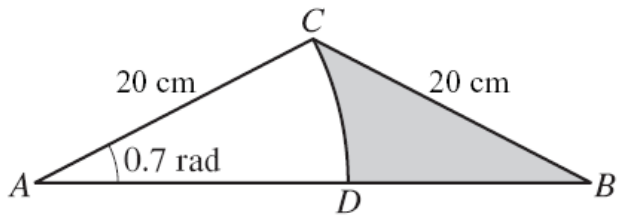
a) Find the area of the shaded segments below



b) The diagram shows OAB is a sector of a circle with centre O and radius 5 cm. BC is 1 cm. Find the area of the shaded segments below



2.4 Solving problems involving area of a circles



The diagram shows an isosceles triangle ABC in which $BC = AC = 20$ cm, and angle $BAC = 0.7$ radians. DC is an arc of a circle, centre A . Find, correct to 1 decimal place,
(i) the area of the shaded region, [4]

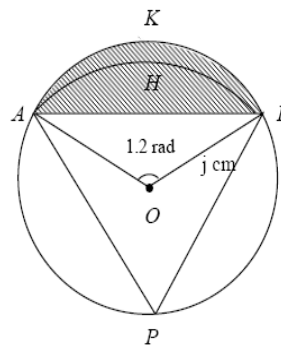


DIAGRAM 2

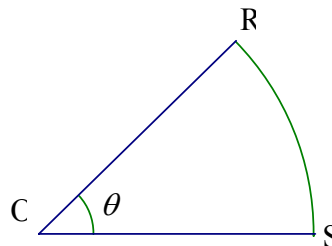
Diagram 2 shows a circle $AKBP$ centred at O , with radius j cm and a sector $APBH$ centred at P with radius 15 cm. Given that the ratio of the arc AHB to the arc AKB is $6:7$ and $\angle AOB = 1.2$ radian. Calculate
(a) the value of j [3 marks]
(b) the area of shaded region. [4 marks]

Homework Text Book Exercise 8.3.3 page 190 and skill practice 8.3 pg 191

Enrichment 1

1 **SPM 2003(P1)**

Diagram shows a sector ROS with center O .

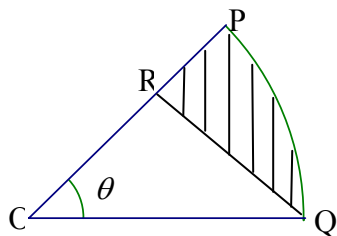


$\theta = 0.8153 \text{ rad}$

The length of the arc RS is 7.24 cm and the perimeter of the sector ROS is 25 cm. Find the value of θ , in rad.
 [3 marks]

2 **SPM 2003(P2)**

Diagram shows the sector POQ , center O with radius 10 cm. The point R on OP is such that $OR : OP = 3 : 5$



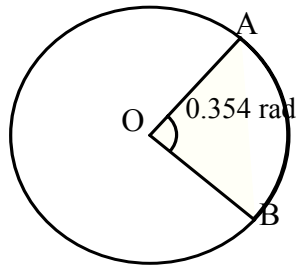
Calculate

- (a)** the value of θ , in rad [3 marks]
- (b)** the area of the shaded region, in cm^2 [4 marks]

(a) $\theta = 0.9273 \text{ rad}$
(b) 22.365 cm^2

3 **SPM 2004 (P1)**

Diagram shows a circle with center O .



Given that the length of the major arc AB is 45.51 cm, find the length, in cm, of the radius. (Use $\pi = 3.142$)
[3 marks]

| |
|------------------------|
| $r = 7.675 \text{ cm}$ |
|------------------------|

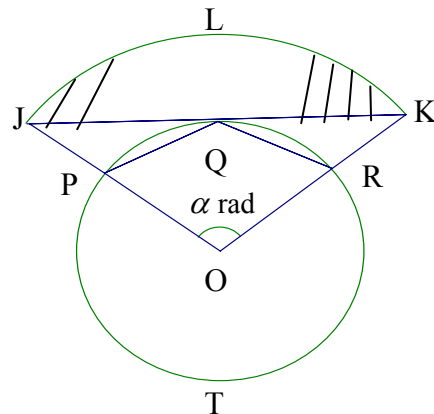
4 **SPM 2004 (P2)**

Diagram shows a circle $PQRT$, center O and radius 5 cm. JQK is a tangent to the circle at Q . The straight lines, JO and KO , intersect the circle at P and R respectively. $OPQR$ is a rhombus. JLK is an arc of a circle, center O .

Calculate

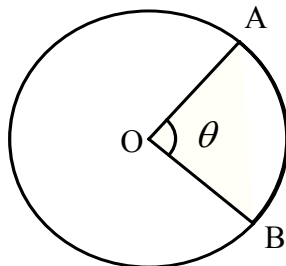
- (a) the angle α , in terms of π [2 marks]
- (b) the length, in cm of the arc JLK [4 marks]
- (c) the area, in cm^2 , of the shaded region. [4 marks]

| |
|---|
| <p>(a) $\alpha = \frac{2}{3}\pi \text{ rad}$</p> <p>(b) $S = 20.94 \text{ cm}$</p> <p>(c) 61.40 cm^2</p> |
|---|



5 **SPM 2005 (P1)**

Diagram shows a circle with center O .



The length of the minor arc AB is 16 cm and the angle of the major sector AOB is 290° . Using $\pi = 3.142$, find

- (a) the value of θ , in radians
(Give your answer correct to four significant figures)
- (b) the length, in cm, of the radius of the circle. [3 marks]

| |
|---|
| <p>(a) 1.222 rad</p> <p>(b) $r = 13.09 \text{ cm}$</p> |
|---|

6 SPM 2005 (P2)

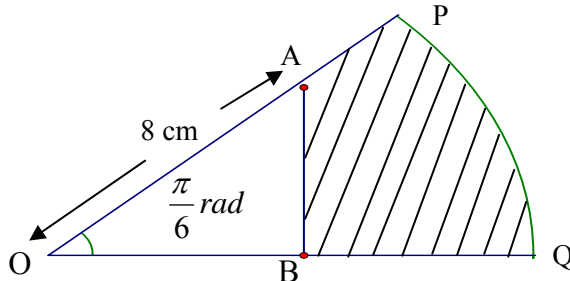
It is given that $OA:OP = 4:7$ (Use $\pi = 3.142$)

Calculate

- (a) the length, in cm of AP
- (b) the perimeter, in cm, of the shaded region,
- (c) the area, in cm^2 , of the shaded region.

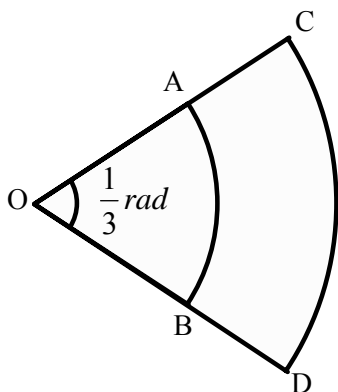
[1 mark]
 [5 marks]
 [4 marks]

(a) 6 cm
 (b) 24.403 cm
 (c) 37.46 cm^2



Enrichment 2

- 1 Diagram shows an arc AB and CD of two sector of a circle with center O. Given that $BD = AC = 3$ cm. If the perimeter of ABDC is 12 cm, find the radius of OB.

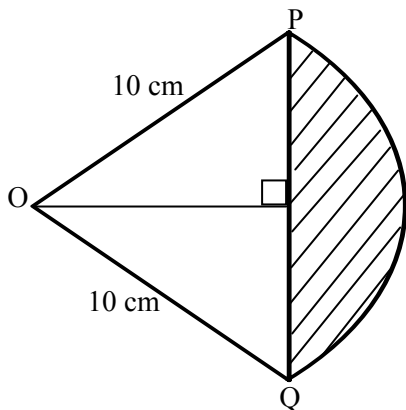


$r = 7.5$ rad

- 2 Diagram shows one part of a circle with center O and radius 10 cm. Given that the length of chord PQ is 6 cm. Find

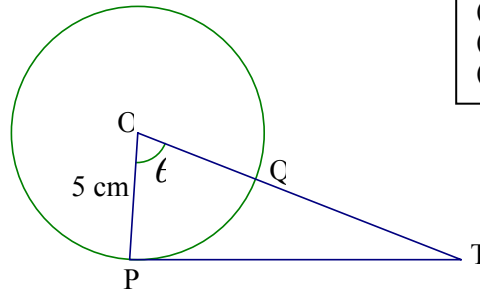
- (a) Angle POQ
- (b) Area of OPQ
- (c) Area of the shaded part

(a) $17^{\circ}27'$
 (b) 28.63 cm^2
 (c) 1.855 cm^2



3 Diagram shows a circle with center O and radius 5 cm. Given that $PT = 5\sqrt{3}$. PT is a tangent of a circle at point P. Find

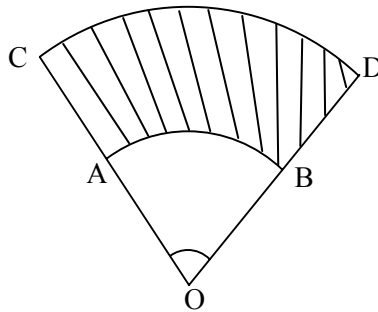
- (a) Angle θ
- (b) Area of a minor sector ,OPQ
- (c) Area of the shaded part.



- | | |
|-----|---------------------------------|
| (a) | $60^\circ = 1.0472 \text{ rad}$ |
| (b) | 13.09 cm^2 |
| (c) | 8.5606 cm^2 |

4 Diagram shows two arc of AB and CD of two circles with center O and radius OA and OD respectively. Given that OA = 10 cm and OD = 14 cm and the perimeter of ABCD is 44 cm. Find

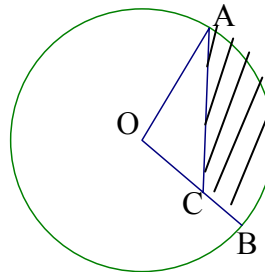
- (a) Angle θ
- (b) Area of ABCD



- | | |
|-----|-------------------|
| (a) | 1.5 rad |
| (b) | 72 cm^2 |

5 The area of a sector AOB of a circle with center O and radius 10 cm is $10\pi \text{ cm}^2$. Point A joint to point C at OB. OC = 6 cm.

- (a) Find the value of angle AOB.
- (b) Find the area of shaded part.

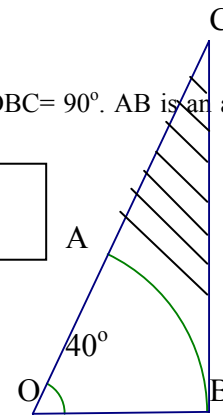


- | | |
|-----|-----------------------------------|
| (a) | $360 / \frac{\pi}{5} \text{ rad}$ |
| (b) | 13.786 cm^2 |

6 Diagram shows a triangle OBC which OB = 3.5 cm, angle AOB = 40° and angle OBC= 90° . AB is an arc of a circle with center O. Find

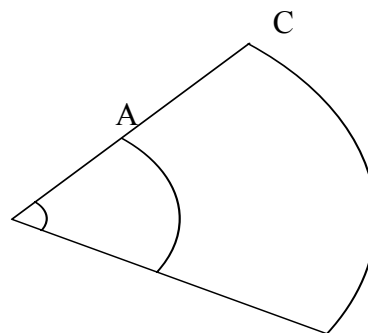
- (a) Perimeter OAB
- (b) Area of shaded part

- | | |
|-----|-----------------------|
| (a) | 9.4434 cm |
| (b) | 0.8637 cm^2 |



7 Diagram shows two sectors of AOB and OCD of a circle with center O and radius $2k$ and $3k$ respectively. Angle AOB is $\frac{3}{4} \text{ rad}$ and area of ABCD is 30 cm^2 . Find

- (a) The value of k
- (b) The difference of length AB and length CD.



- | | |
|-----|---------|
| (a) | $k = 4$ |
| (b) | 3 |