module

module

TEACHING & LEARNING

ADDITIONAL MATHEMATICS

FORM 4

DIFFERENTIATION

CHAPTER 9

NAME:……………………………………………………….

FORM :……………………………………………………

Date received : ………………………………

Date complete …………………………..

Marks of the Topical Test : ……………………………..

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CALCULUS

1 $y = uv$, $\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$

2. $y = \frac{u}{v}$, $\frac{dx}{dy} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$,

3 $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

FROM CS

Students will be able to:

1. Understand and use the concept of gradients of curve and differentiation.

1.1 Determine the value of a function when its variable approaches a certain value.
1.2 Find the gradient of a chord joining two points on a curve.
1.3 Find the first derivative of a function *y* = *f*(*x*), as the gradient of tangent to its graph.
1.4 Find the first derivative of polynomials using the first principles.
1.5 Deduce the formula for first derivative of the function *y* = *f*(*x*) by induction.

1.1 Determine the value of a function when its variable approaches a certain value

· $\underset{x \to a}{\text{limit}}$ f(x) is the value of f(x) when x approaches the value of a

Example 1

Find the limit for each question below

(a) $\lim_{x\to 0}(0.2)^x =$	(b) $\lim_{x\to\infty}(0.2)^x =$	c) $\lim_{x\to 2}(0.2)^x =$	d) $\lim_{x\to -2}(0.2)^x =$
e) $\lim_{n\to 0}\frac{1}{3+n^2-n} =$	(f) $\lim_{n\to\infty}(\frac{1}{3})^n =$	(g) $\lim_{x\to\infty}\frac{4x}{1+x^2} =$	h) $\lim_{n\to -3}\frac{1}{3+n^2-n} =$

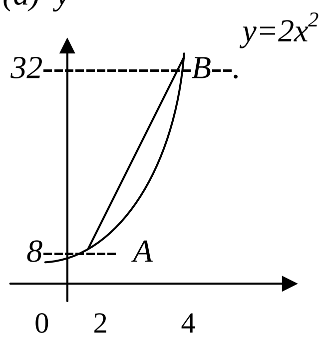
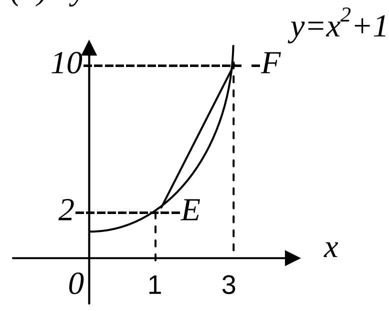
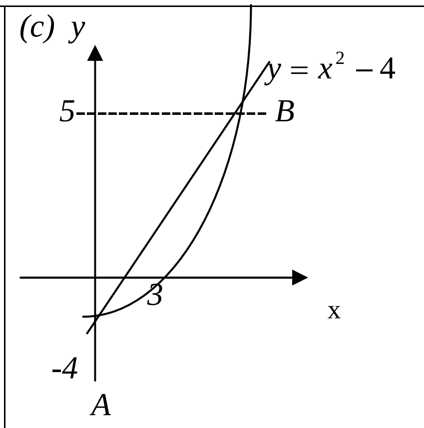
Exercises 1 : Find the limit for each question below

a) $\lim_{n\to 0}\frac{1+2n}{2+n}$	b) $\lim_{n\to\infty}\frac{1+2n}{2+n}$	c) $\lim_{n\to 2}\frac{1+2n}{2+n}$
d) $\lim_{x\to 0}\frac{n^2-4}{n-2}$	e) $\lim_{n\to 2}\frac{n^2-4}{n-2}$ (SPM97)	f) $\lim_{x\to\infty}\frac{n^2-4}{n-2}$
g) $\lim_{x\to\infty}\frac{2x^2}{3x^2+4x+1}$	h) $\lim_{x\to\infty}\frac{4n^3}{43n+5n^3}$	I) $\lim_{x\to -3}\frac{x^2+x-6}{x+3}$

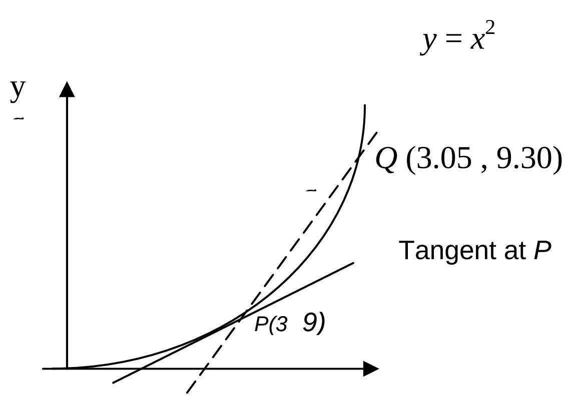
Homework : Text Book Exercise 9.1.1page 198

1.2 Find the gradient of a chord joining two points on a curve

Example 2 : Determine the gradient of the cord AB as shown below

(a)	(b)	(c)
		

1.3 Finding the first derivative of a function $y = f(x)$, as the gradient of tangent to its graph.



Complete the table below and hence determine the first derivative of the function $y = x^2$

Point Q (x_2, y_2)		$x_2 - x_1$	$y_2 - y_1$	Gradient PQ
x_2	y_2			
3.05	9.30			
3.01				
3.001				
3.0001				

When $Q \rightarrow P$, gradient $PQ \rightarrow 6$, then the gradient at point $P = 6$

so , the gradient of the tangent at $P = \lim_{\delta x \to 0} \frac{\delta y}{dx} = \frac{dy}{dx} = 6$ at $x = 3$

.

1.4 Find the first derivative of polynomials using the first principles

The first derivative, $\frac{dy}{dx}$ of the polynomial y = ax^n + bx^{n-1} + ……….+ c where a b c and n are constants can be determined as shown below.

a) Let δx be a small change in x and δy be a small change in y.

b) Substitute x and y in the equation y = f(x) with x + δx and y + δy respectively

c) Express δy in terms of x and δx

d) Find $\frac{\delta x}{\delta y}$ and then, $\lim_{\delta x \to 0} \frac{\delta y}{\delta x} = \frac{dy}{dx}$.

Example 1

a) Determine the first derivative of y = x^2 + 4 by using the first principle x Solution	*b)* Determine the first derivative of y = $\frac{1}{x}$ by using first principle x *Solution :*

Exercise 1 :

1. Given y = $3x^2$ + 5,find $\frac{dy}{dx}$ by using the first principle (SPM 94) Solution Homework : Text Book Exercise 9.1.4 page 201	2. Determine the first derivative of y = $\frac{4}{x} - 3$ by using first principle SPM (97) Solution :

1.5 Deduce the formula for first derivative of the function $y = f(x)$ by induction
Complete the table below by using the first principle.

Function	Derivative
$3x$	
x^2	
$4x^2$	

From the pattern, a formula of the first derivative of ax^n can be deduced which is $\frac{d}{dx}(ax^n) = nax^{n-1}$

FROM CS

Students will be able to:

2. Understand and use the concept of first derivative of polynomial functions to solve problems

2.1 Determine the first derivative of the function $y = ax^n$ using formula.
2.2 Determine value of the first derivative of the function $y = ax^n$ for a given value of x.
2.3 Determine first derivative of a function involving:
a) addition, or b) subtraction of algebraic terms.
2.4 Determine the first derivative of a product of two polynomials.
2.5 Determine the first derivative of a quotient of two polynomials.
2.6 Determine the first derivative of composite function using chain rule.
2.7 Determine the gradient of tangent at a point on a curve.
2.8 Determine the equation of tangent at a point on a curve.
2.9 Determine the equation of normal at a point on a curve.

***2.1* Determining the first derivative of the function $y = ax^n$ using formula**

1. If $y = k$, where k is a constant then or $\frac{dy}{dx} = \frac{d}{dx}(k) = 0$
2. If $y = ax^n$ where k is a constant and n is positive and negative integer then $\frac{dy}{dx} = \frac{d}{dx}(ax^n) = nax^{n-1}$
3. If $f(x) = ax^n$, then $f'(x) = anx^{n-1}$. Notation $f'(x)$ is read as f prime x

Example 2: find $\frac{dy}{dx}$ for each of the following.

(a) $y = 2x$	(b) $y = x$	(c) $y = 3x^2$	d) $y = -4x$	e) $y = \frac{3}{x^4}$

Exercise 2: find $\frac{dy}{dx}$ for each of the following.

a) $y = -5$	b) $y = \frac{3}{5}x^3$	c) $y = -6x^3$	d) $y = \frac{1}{2x^2}$	e) $y = \frac{3x^3}{12}$

Homework : Text Book Exercise 9.2.1 page 204

2.2 Determining value of the first derivative of the function $y = ax^n$ for a given value of x.

Example3**:** Find the value of $\frac{dy}{dx}$ at each of the given value of x

(a) $y = 15x^3$ *when* $x = -1$	*(b)* $f(x)= \frac{1}{2x^2}$ *, find* $f'(2)$	*(c)* $y=\frac{4}{x^3}$ *when* $x=0.5$	*(d)* $f(t) = 3t^3$*, find* $f(-3)$

Exercise 3**:** Find the value of $\frac{dy}{dx}$ at each of the given value of x

(a) $y = -2x^3$ *when* $x = -1$	*(b)* $f(x)= \frac{3}{2x^3}$ *, find* $f'(2)$	*(c)* $y = \frac{2}{x^4}$ *when* $x=0.5$	*(d)* $f(t) = 5t^4$*, find* $f(-3)$

Homework : Text Book Exercise 9.2.2 page 205

2.3 Determining first derivative of a function involving:

a) addition, or b) subtraction of algebraic terms.

If $f(x) = p(x) + q(x)$ *then* $\frac{d}{dx}[f(x)] = \frac{d}{dx}[p(x)] + \frac{d}{dx}[q(x)]$ *or* $f'(x) = p'(x) + q'(x)$

Example4 : Find $\frac{dy}{dx}$ for each of the following

(a) $y = 3x^4 + 5x^5 + 1$	*(b)* $y = (2x - 3)(x^2 + 5)$	*(c)* $f(x) = \frac{6x^3 + 7x - 4}{x}$	d) $y = \frac{2}{5}x^5 + \frac{3}{4}x^4$

Exercise4. Find $\frac{dy}{dx}$ for each of the following

a) $y = 5x^2 + 3x^3 + 2x - 1$	*b)* $\sqrt{t} + 3t^3 - 6t^2 + 7$	*c)* $t^{\frac{4}{5}} + t^{\frac{3}{2}} - 5t^2$	*d)* $7s^7 + 5s^3 - 8s$

Homework : Text Book Exercise 9.2.3 page 206

2.4 Determining the first derivative of a product of two polynomials

Let y, u and v be the functions of x. If $\mathbf{y = u\,v}$ then $\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$ or $y' = uv' + vu'$.

It is called the product rule.

Example 5 : Given that $y = x^3(3 - 5x^3)$, find $\frac{dy}{dx}$

Example 6 : Differentiate $(4x^3 + 3)(x^2 - 2x)$ with respect to x

Solution	*Solution*

Example 6 : Find $\frac{dy}{dx}$ for each of the following function.

a) $x^4(1 + x^7 + 6x^4)$ (SPM 96)	b) $y = (x^5 + 3x^4 + 2x)(x^7 + 7x^2 + 6)$	c) $\sqrt{x}(4x - 6x^3 + 3)$

Exercise 6 : Find $\frac{dy}{dx}$ for each of the following function

a) $y = (2x+1)(x^3 - x^2 - 2x)$	(b) $(x^3 - 4x)(x^2 - \frac{1}{x})$	c) $(1 - \frac{1}{x})(2x + x^2 - \frac{1}{x^2})$
d) $(x^6 - 1)(3x^2 - 5x^3)$	e) $\sqrt{x^3}(x^2 + 4x - 1)$	f) $(\frac{1}{\sqrt{x}} - 2x)(\frac{1}{x^3} - 4x + 3)$

Homework : Text Book Exercise 9.2.4 page 208

2.5 Determining the first derivative of a quotient of two polynomials

Let y, u and v be the function of x **y=** $\frac{u}{v}$ then $\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$ **=** ***or*** $\frac{dy}{dx}$ **=** $\frac{vu' - uv'}{v^2}$

It is called the quotient rule

Example 7 : Find $\frac{dy}{dx}$ for each of the following functions

1. $y = \frac{x^4 + x}{4 + x^3}$	*2.* $y = \frac{\sqrt{t} + 2t - 1}{t^{\frac{3}{2}}}$
*3.*Given that $f(x) = \frac{1 - 2x^2}{4x - 3}$ *find f ' (SPM 93)*	*4.*Given that $f(x) = \frac{1 - 2x^3}{x - 1}$ *find f '(x) (SPM 95)*

Exercise 7

1. If $y = \frac{(1-x)(1+x)}{(x-3)(3+x)}$ find $\frac{dy}{dx}$	*2. Given that $F(x) = \frac{1-2x^3}{x+2}$ find $F'(x)$*
*3.*Differentiate $\frac{2x^3+3}{x^2}$ with respect to *x*	4. Differentiate $\frac{2t}{t^2-4}$ with respect to *t*

Homework : Text Book Exercise 9.2.5 page 210

2.6 Determining the first derivative of composite function using chain rule

> If a composite function is in the form $y = u^n$ where u = f(x) and n is an integer, then by the chain rule
>
> $$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}.$$

Example 8

1. Differentiate $(4x^2 - 3)^5$ with respect to x	2. Differentiate $\frac{3}{(t^2-3t)^2}$ with respect to *t*

Exercise 8

1. Find $\frac{d}{dx}(\frac{1}{2x-1})$ SPM94	2.Find y ' if y = $\frac{4}{3x+2}$ SPM 98
3. Differentiate $x^4(1+3x)^7$ with respect to *x* SPM 96	4 Given that f(x) = 4x $(2x - 1)^5$ find f '
5. Differentiate $\frac{x^2}{(2x+1)^2}$ with respect to x	6. Differentiate $\frac{(t^2-2)^3}{(4t^2-5)^2}$ with respect to *t*

Homework : Text Book Exercise 9.2.6 page 212

2.7 Determining the gradient of tangent at a point on a curve

Notes:

> a) $\frac{dy}{dx}$ represents the gradient of the tangent of the curve y = f(x)
>
> b) The gradient of the tangent at a point A (p,q) on a curve y = f(x) can be determined by substituting $x = p$ into $\frac{dy}{dx}$

Example 9 : Find the gradient of a tangent at the given point for each of the following curves

(a) $y = 4x^2 - 6x + 1$, (2,5)	(b) $y = x^2 - 2x$, (-1,1)	(c) $y = \frac{6 - 4x}{x}$, (-2,7)

Exercise 9 : Find the gradient of a tangent at the given point for each of the following curves

(a) $y = 3x^2 - 7x + 3$, (3,5)	(b) $y = 2x^3 - 3x$, (-2 ,1)	(c) $y = \frac{6 - 3x}{x^2}$, (-1,7)

Homework : Text Book Exercise 9.2.7 page 213

2.8 Determining the equation of tangent at a point on a curve

Notes:

> The equation of the tangent at point $P(x_1,y_1)$ on the curve $y = f(x)$ can be determined by
> (a) Finding the gradient, m, of the tangent at point P
> (b) Use the formula $y - y_1 = m(x - x_1)$ to find the equation of the tangent.

Example 10 : Find the equation of the tangent for each of following equations and the corresponding points.

(a) $y = 2x^2 - 3x + 4$ at point (2,3)	(b) $y = x^3 + 3x^2$ at point (-1, 2)

Example 10 : Find the equation of the tangent for each of following equations and the corresponding points.

(a) $y = 3x^2 - 2x - 4$ at point (3, 17)	(b) $y = 3x^3 + 2x^2$ at point (-1, -1)

Homework : Text Book Exercise 9.2.8 page 214

2.9 Determining the equation of normal at a point on a curve

Notes

> The equation of the normal to the curve $y = f(x)$ at point $P(x_1, y_1)$**,** can be determined by
> (a) Finding the gradient ,m_1,of the tangent at point *P*,
> (b) Finding the gradient, m_2, of the normal at point P for which $m_1 m_2 = -1$
> (c) using the formula $y - y_1 = m_2 (x - x_1)$ to find the equation of the normal

Example 11 : Find the equation of the normal for each of following equations and the corresponding points.

(a) $y = x^2 - 4x + 1$ at point (3,-2)	*(b)* $y = 2x + \frac{8}{x}$ at point (1,10)

Exercise 11 : Find the equation of the normal for each of following equations and the corresponding points.

(a) $y = x^2 - 2x - 4$ at point (3,-1)	*(b)* $y = \frac{x}{x-5}$ at x = 7

Homework : Text Book Exercise 9.2.9 page 215

Example 12 : [Ans 1 a) 14 b) y = 14x -23 , 14y = -x + 72]

1. Given $y = (x^2 + 1)(x - 1)^2$ find (a) the gradient of the curve for which $x = 2$ (b) the equation of the tangent and the normal at $x = 2$ Solution (a) b)	2.Given f(x) = $\frac{x+4}{x-1}$ find the value of x if $f'(x) = -\frac{5}{4}$ and hence , find the equation of the tangent and normal at that point..

Exercise 12 :

1. Find the equation of the tangent on a curve $y = (x^2 + 1)(2x + 5)$ at point (-1,6)	2.The gradient of the normal on the curve $y = (2x - 3)^2$ is $\frac{1}{2}$. Find the x-coordinates of the point.

3. The curve $y = hx + \frac{k}{x^2}$ has the gradient of 2 at $(-1, -\frac{7}{2})$ Find the values h and k . (SPM 96)	*4.* Find the gradient of the curve $y = \frac{4}{3x+2}$ at point (-2, -1). Hence , find the equation of the normal . (SPM 98)
5 Find the equation of the tangent on a curve *$y = 3x^2 - 4x + 2$ which is parallel to the line $y = 2x + 5$*	*6. Given that $f(x) = x - \frac{1}{x}$. Find the positive value of x for which $f'(x) = 2$, Hence, find the equation of the tangent and normal.*

Homework : Skill Practice 9.2 Page 216

FROM CS

Students will be able to:

6. Understand and use the concept of second derivative to solve problems

6.1 Determine the second derivative of function $y = f(x)$.

6.2 Determine whether a turning point is maximum or minimum point of a curve using the second derivative

6.1 Determine the second derivative of function $y = f(x)$.

> If: $y = f(x)$, then $\frac{dy}{dx} = f'(x)$ and $\frac{d}{dx}\left|\frac{dy}{dx}\right| = \frac{d^2y}{dx^2} = f''(x)$ is called the second derivative of $y = f(x)$

Example 13:

1. Given that $y = 4x^3 + 2x - \frac{5}{x}$ find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$	2 . Given that $f(x) = (1-2x^2)^4$ .Find $f''(x)$ and $f''(0)$

Exercise 13 :

1. Given that $y = (1+x^2)^4$ find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$	2. Given that $y = \frac{1}{x} + \frac{2}{x^2}$ find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$
3.Find f'' and $f''(2)$ for $f(x) = 2x^4 - x^3 + 15x + 1$	4. Given $f(x) = (1 - x^2)^3$ Find f' and f''

5 .Given that $y = x(3-x)$, Express $y\frac{d^2y}{dx^2} - x\frac{dy}{dx} + 12$ in term x. then solve $y\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 12$ (SPM 95)	6. Given that $f(x) = (2x - 3)^5$ Find f'' (SPM 97)

Homework : Text Book Exercise 9.6.1 page 231

FROM CS

Students will be able to:

3. Understand and use the concept of maximum and minimum values to solve problems

3.1 Determine coordinates of turning points of a curve.
3.2 Determine whether a turning point is a maximum or a minimum point.
3.3 Solve problems involving maximum or minimum values.

3.1 Determining coordinates of turning points of a curve

Notes :

A curve $y = f(x)$ has a turning point or stationary point where $\frac{dy}{dx} = 0$. At this point the tangent is parallel to the x- axis

Example 14. Find the coordinates of the turning point of the following curve

a) $y = 8 - x^2$	b) $y = 3x - x^3$
c) $y = x + \frac{4}{x} - 2$	d) $y = \frac{4x^2 + 9}{x}$

Homework : Text Book Exercise 9.3.1 page 219

3.2 Determining whether a turning point is maximum or minimum point of a curve using the second derivative

3.3 Determining whether a turning point is a maximum or a minimum point

Notes :

> The types of turning points of a curve y = f(x), can be tested as follows
>
> a) find $\frac{dy}{dx}$ of the curve $y = f(x)$. then find the value of x for which $\frac{dy}{dx} = 0$
>
> b) Find the corresponding value of y by substituting the value of x into y = f(x)
>
> c) Determine whether a turning point is maximum or minimum point by substituting the values of x into $\frac{d^2y}{dx^2}$. If $\frac{d^2y}{dx^2} > 0$ the point is minimum point or if $\frac{d^2y}{dx^2} < 0$ the point is maximum point.

Example 15 .

1. Find the coordinates of turning point of the curve $y = 12x - 3x^2$.Hence determine whether the turning point is maximum point or minimum point.	2. Show that the curve $y = 5x^2 - 5x + 1$ has only one turning point. Hence determine whether the turning point is maximum point or minimum point

Exercise 15

1. . Show that the curve $y = 8x + \frac{1}{2x^2}$ has only one turning point. Hence determine whether the turning point is maximum point or minimum point *[Ans* :(1/2,6)min]	2. Find the coordinates of turning point of the curve $y = \frac{x^3}{3} - \frac{x^2}{2} - 6x$. Hence determine whether the turning point is maximum point or minimum point. [Ans (3,-27/2)min, (-2,22/3)max]

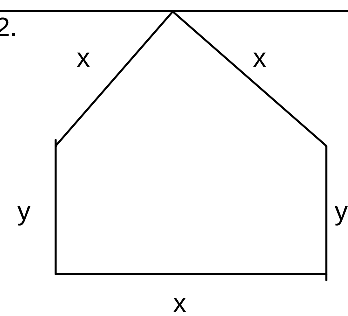
Homework : Text Book Exercise 9.3.2 page 221

3.3 Solve problems involving maximum or minimum values.

Example 16 ;

1. A piece of wire of length 60 cm is bent to form a rectangle. Find the dimensions of the rectangle for which the area is a maximum. [Ans 15 cm]	2. A solid cylinder with a x radius has a volume of 800cm^3 a) Show that the total surface area, A cm^2 . of the cylinder is given by $A = 2\pi x^2 + \dfrac{1600}{x}$ b) Hence, find the value of x which makes the surface area a minimum [Answer x = 5

Exercise 16

1. Given a rectangle of area 12 cm^2 and length x cm express the perimeter, *P*, of the rectangle in terms of x . Hence find the minimum value of *P* . [Ans 2x +24/x, 8$\sqrt{3}$]	2.  The diagram shows a pentagon ABCDE. If the perimeter of the pentagon is 18 cm, find the values of x and y when the area is a maximum [x= 4.22, y=2.67]

Homework : Text Book Exercise 9.3.3 page 223

FROM CS

Students will be able to:

4. Understand and use the concept of rates of change to solve problems.

4.1 Determine rates of change for related quantities

4.1 Determining rates of change for related quantities

Notes :

If y is a function of x and x is a function of time, t , then the link between y and t can be determine by chain rule . $\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$. If the rate of change is positive, it means increment and if the rate of change is negative, it means decrement.

Example 16 .

a)The radius, r cm of a circle is 20 cm and it is increasing at the rate of 3cms^{-1}. At what rate is the area of the circle increasing.	2. The area of a circle is decreasing at the rate 2 cm^{2}s $^{-1}$ How fast is the radius decreasing when the area is 32π cm^2

Exercise 16 .

a)The radius, r cm of a circle is 10 cm and it is increasing at the rate of 4 cms^{-1}. At what rate is the area of the circle increasing.	2. Two variable, x and y are related by equation y = $3x - \frac{5}{x}$ If y is changing at a rate of 11.5 units per seconds , find the rate of change of x when x = -2

Homework : Text Book Skill Practice 9.4 Pg 226

Students will be able to:

5. Understand and use the concept of small changes and approximations to solve problems.

5.1 Determine small changes in quantities.

5.2 Determine approximate values using differentiation

5.1 Determining small changes in quantities

Notes :

1. Given a function y = f(x), then as x makes a small change, y will also change by a small quantity
2. The small change in x and y are denoted by δx and δy respectively
3. If the value of δx and δy are positive its means a small increment in x and y respectively while If the value of δx and δy are negative its means a small decrement in x and y respectively.

Example 16*:*

1. If $y = 3x^2 + 5x + 2$, find the small change in y when x change from 3 to 3.02	2.Find the approximate change in volume of a sphere if its radius increase from 5 cm to 5.02

Exercise 16

1 Given that $y = 2x - x^2$, Find a) the small change in y when x change from 2 to 2.01 [Ans= - 0.02] b) the small change in y when x change from 2 to 1.99 [Ans = 0.02]	2. Given that $y = 6x^3 + x^2$, find the small change in y when x decreases from 3 to 2.99

5.2 Determining approximate values using differentiation

1. If y = f(x) then $\frac{\delta y}{\delta x} \approx \frac{dy}{dx}$ where δx is small amount of change in x δy is small change in y

Example 17

1 Given that y = $\frac{24}{x^4}$, find the approximate value of $\frac{24}{(2.02)^4}$ [Ans 1.44]	2.Given that y = $9x^{\frac{2}{3}}$, Find $\frac{dy}{dx}$ when y = 36. Hence find the small change in x when y increases 36.0 to 36.3 [Ans 3, 0.1]

Exercise 17

1. Given that y = $\frac{16}{x^4}$, find $\frac{dy}{dx}$ when x = 2. Find the approximate value of $\frac{16}{(1.98)^4}$ [Ans : -2 , 1.04]	2.Given that y = $\frac{1}{\sqrt[3]{x}}$. Find the approximate value of $\frac{1}{\sqrt[3]{0.9}}$ [Ans $1\frac{1}{30}$]

SPM Questions

SPM 2003 P 2

a) Given that $y = x^2 + 5x$, use differentiation to find the small change in y when x increases from 3 to 3.01. [Answer 0,11] [3 marks]

b) Given that $y = 14x(5 - x)$, calculate

i) the value of x when y is a maximum

ii) the maximum value of y [3 marks]

SPM 2003 P2

a) Given that $y = x^2 + 2x + 7$ find the value of x if

$x^2 \frac{d^2y}{dx^2} + (x - 1)\frac{dy}{dx} + y = 8$. [Answer $x = 5/3$ or $x = -1$] [4 marks]

b)

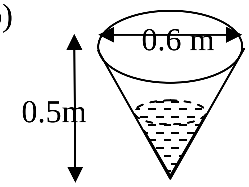


Diagram 2

Diagram 2 shows a conical container of diameter 0.6 m and heights 0.5 m. Water is poured into the container at a constant rate of 0.2 $m^3 s^{-1}$ { use $\pi = 3.142$, Volume of a cone = $\frac{1}{3}\pi r^2 h$

[4 marks]

SPM 2004 P 1

a) Differentiate $3x^2(2x - 5)^4$ with respect to x. [Answer $6x(6x-5)(2x-5)^3$] [3 marks]

b) Two variables , x and y are related by the equation $y = 3x + \frac{2}{x}$. Given that y increases at a constant rate of 4 units per seconds, find the rate of change of x when x = 2 [3 marks]

SPM 2004 P 2

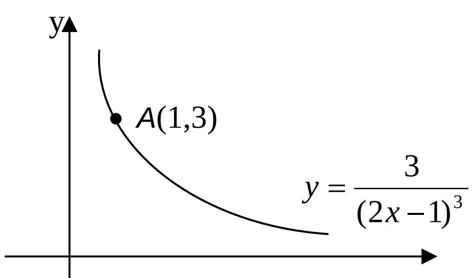


Diagram 5

Diagram 5 shows a part of the curve $y = \frac{3}{(2x-1)^3}$, which passed through A(1,3) . Find the equation of the tangent to the curve at the point A

[4 marks]

SPM 2005

3472/1 a) Given that $h(x) = \frac{1}{(3x-5)^2}$ evaluate $h''(1)$ [4 marks]

b) The volume of water, V cm^3, in a container is given by $V = \frac{1}{3}h^3 + 8h$, where h cm is the heights of the water in the container. Water is poured into container at the rate of 10 $cm^3 s^{-1}$. Find the rate of change of the heights of the water in cm s^{-1} , at the instant when its heights is 2 cm. [3 marks]