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TEACHING & LEARNING

ADDITIONAL MATHEMATICS

FORM 4

INDICES AND LOGARITHMS

CHAPTER 5

NAME:……

FORM :……

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Marks of the Topical Test : ……..

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For Internal Circulations Only

Formulae

a) $a^m \times a^n = a^{m+n}$

b) $a^m \div a^n = a^{m-n}$

c) $(a^m)^n = a^{nm}$

d) $\log_a mn = \log_a m + \log_a n$

e) $\log_a b = \dfrac{\log_c b}{\log_c a}$

f) $\log_a \dfrac{m}{n} = \log_a m - \log_a n$

g) $\log_a m^n = n \log_a m$

FROM CS

Students will be able to:

1. Understand and use the concept of indices and laws of indices to solve problems.

1.1 Find the value of numbers given in the form of:
   a) integer indices.
   b) fractional indices.

1.2 Use laws of indices to find the value of numbers in index form that are multiplied, divided or raised to a power.

1.3 Use laws of indices to simplify algebraic expressions.

1.1 Finding the value of numbers given in the form of:

a) integer indices. $a^n = \underbrace{a \times a \times a \times a \ldots\ldots a}_{n \text{ times}}$

b) fractional indices ; $a^{\frac{1}{n}} = \sqrt[n]{a}$; $a^{\frac{m}{n}} = \sqrt[n]{a^m}$

Example 1

Find the values each of the following

i) $0.2^3 =$	ii) $\left(\frac{1}{5}\right)^4 =$	iii) $3^6 =$	iv) $\left(-\frac{2}{3}\right)^4 =$
v) $\left(\frac{8}{125}\right)^{\frac{2}{3}} =$	vi) $\left(\frac{9}{64}\right)^{\frac{1}{2}} =$	vii) $\left(\frac{8}{27}\right)^{\frac{1}{3}} =$	vii) $27^{\frac{4}{3}} =$

Exercises 1

a) $10^3 =$	b) $\left(-\frac{1}{3}\right)^3 =$	c) $(4.5)^3 =$	d) $\left(\frac{1}{4}\right)^2 =$	e) $(5^{-1})^4 =$
f) $(0.125)^{\frac{1}{3}} =$	b) $\left(\frac{32}{243}\right)^{-\frac{3}{5}} =$	c) $(0.16)^{\frac{1}{2}} =$	d) $27^{\frac{4}{3}}$	e) $4^{-\frac{3}{2}}$

Homework : Text Book Exercise 5.1.1 page 71

1.2 Using laws of indices to find the value of numbers in index form that are multiplied, divided or raised to a power

Laws of indices

a) $a^m \times a^n = a^{m+n}$ **b)** $a^m \div a^n = a^{m-n}$ **c)** $(a^m)^n = a^{mn}$

Example 2
Simplify the following indices:

(i) $a^2 \times a^5 =$	(ii) $(p^6 q^{-2})^{\frac{3}{2}}$	(iii) $\dfrac{x^{6n}}{x^{2n}}$

Exercises 2
Simplify the following indices: [Ans a^8 b) $a^3 b^{-9}$ c) $b^{2n/3-m/2}$ d) $b^{2n-3/2m}$]

(a) $a^6 \times a^2$	(b) $(a^4 b^{-12})^{3/4}$	(c) $(b^{\frac{8n}{3}} \div b^{2m})^{1/4}$	(d) $(b^{\frac{8n}{3}} \div b^{2m})^{\frac{3}{4}}$

Homework : Text Book Exercise 5.1.2 page 73

Example 3: Simplify each of the following :

(i) $\dfrac{a^{2-n} \times a^{3n}}{a^{2n+5} \times a^4}$	(ii) $2^{3n+1} \times 4 \div 8^{n+1}$	(iii) $5^{2n} \times 5^{2-3n}$

Exercises 3 : Simplify each of the following [Ans a) $2a^{\frac{1}{3}}b^2$ b) 2^{6-4n} c) 5^{6n+4}]

(a) $(2ab^2)^3 \div (8a^4b^6)^{\frac{2}{3}}$	(b) $4^n \times 8^{2-n} \div 2^{3n}$	(c) $5^{3n+5} \div 5^{2-n} \times 5^{1+2n}$

Example 4 : Show that :

(a) $5^n + 5^{n+1} + 5^{n+2}$ is divisible by 31 for all positive integers of *n*	(b) $11^{n+2} + 11^{n+3}$ is divisible by 12 for all positive integers of *n*	(c) $3^n + 3^{n+1} + 3^{n+2}$ is divisible by 31 for all positive integers of *n*

Homework : Text Book Exercise 5.1.3 page 74

FROM CS

Students will be able to:

2. Understand and use the concept of logarithms and laws of logarithms to solve problems
   - 2.1 Express equation in index form to logarithm form and vice versa.
   - 2.2 Find logarithm of a number.
   - 2.3 Find logarithm of numbers by using laws of logarithms.
   - 2.4 Simplify logarithmic expressions to the simplest form.

2.1 **Expressing equation in index form to logarithm form and vice versa**.

Definition of logarithm

If a is a positive number and $a \neq 1$, then $N = a^x \Leftrightarrow \log_a N = x$

($\log_a N$ is read as logarithm of N to the base of a)

Note :

a) $a^0 = 1$ $\quad \log_a 1 = 0$ (The logarithms of 1 to any positive number base is zero)

b) $a^1 = a$ $\quad \log_a a = 1$ (The logarithms of any positive number a to the base a is 1)

Example 5 : Convert the following to logarithm form

(a) $b^{-2} = 0.01$	(b) $c = 64^{\frac{1}{3}}$	(c) $81 = 3^3$

Exercises 5 : Convert the following to logarithm form

(a) $64 = 4^3$	(b) $3^b = \frac{1}{9}$	c) $f = (\frac{2}{3})^{-2}$

Example 6 : Convert the following to index form.

(a) $\log_a 64 = 3$	(b) $\frac{1}{2} = \log_x 9$	c) $\log_a 5 = p$

Exercises 6: Convert the following to index form

(a) $\log_x \frac{1}{9} = -2$	(b) $4 = \log_5 625$	c) $\log_{10} 100 = 2$

Example 7: Solve the following equations ;

(a) $\log_x 3 = 5$	(b) $\log_x 4 = \frac{1}{2}$	(c) $\log_x 4 = 2$	d) $\log_5 x = 0$

Exercises 7: Solve the following equations { Ans a) a b) 125 c) 4 d) 3

(a) $\log_3 x = 2$	(b) $\log_x \frac{1}{5} = -\frac{1}{3}$	c) $\log_x \frac{1}{16} = -2$	d) $\log_x 27 = 3$

Homework : Text Book Exercise 5.2.1 page 76

2.2 **Finding Logarithm of a Number**.

Logarithms to the base of 10 are called **common logarithms** . The values of common logarithms can be determined using a scientific calculator.5

Logarithm of negative numbers and zero are undefined Eg $\log_6 0$ and $\log_{10}(-8)$ are undefined .

If $\log_{10} N = x$, then antilog $N = 10^x$

Example 8 Use a calculator to evaluate each of the following

(a) $\log_{10} 16$	(b) $\log_{10}(\frac{2}{3})^3$	(c) antilog 0.1383	(d) antilog(-0.279)

Exercises 8 : Use a calculator to evaluate each of the following

(a) $\log_{10} 61$	(b) $\log_{10}(\frac{4}{3})^3$	(c) antilog 1.1383	(d) antilog(-0.979)

Example 9 Find the value of the following logarithms

(a) $\log_4 16$	(b) $\log_3(\frac{81}{27})$	(c) $\log_5 125$	(d) $\log_6 216$

Exercises 9 Find the value of the following logarithms

(a) $\log_2 64$	(b) $\log_3 81$	(c) $\log_4 1024$	(d) $\log_5(\frac{1}{625})$

Homework : Text Book Exercise 5.2.2 page 77

2.3 Finding Logarithm of Numbers by Using Laws of logarithms

Laws of Logarithms

(i). $\text{Log}_a xy = \log_a x + \log_a y$ (ii) . $\text{Log}_a(\frac{x}{y}) = \log_a x - \log_a y$

(iii) $\text{Log}_a x^n = n \log_a x$

Example 10: Evaluate each of the following without using a calculator Ans : a) 4 b) –2 c) 0 d) $\frac{1}{3}$

(a) $\log_2 16$	(b) $\log_5 \frac{1}{25}$	(c) $\log_a 1$	(d) $\log_c \sqrt[3]{c}$

Exercises 10: Evaluate each of the following without using a calculator Ans : a) 4 b) -2 c) 3 d) $\frac{1}{3}$

(a) $\log_3 81$	(b) $\log_2 \frac{1}{4}$	(c) $\log_{0..5}\ 0.125$	(b) $\log_{27}\ 3$

Example 11: Ans; a) 2 b) i)1011 ii)025 iii)2.65

a) Find the value of $\log_2 7 + \log_2 12 - \log_2 21$	b) Given that $\log_5 2 = 0.43$ and $\log_5 3 = 0.8$ evaluate the following i) $\log_5 6$ ii) $\log_5 1.5$ iii) $\log_5 72$

Exercises 11 : A) Evaluate each of the following Ans : a)3 b)2 c)2

(a)$\log_3 21 + \log_3 18 - \log_3 14$	(c) $2\log_4 2 - \frac{1}{2}\log_4 9 + \log_4 12$	(a) $\log_8 45 - \frac{1}{2}\log_8 81 + 7\log_8 2 - \log_8 10$

B) Given that $\log_7 4 = 0.712$ and $\log_7 5 = 0.827$. Evaluate each of the following
[Ans : a)0.115 b)1.712 c)2.366 d) –0.712]

(a) $\log_7 1\frac{1}{4}$	(b) $\log_7 28$	(c) $\log_7\ 100$	(b) $\log_7\ 0.25$

Homework : Text Book Exercise 5.2.3 page 78

2.4 Simplifying logarithmic expressions to the simplest form.

Example 12 : Express each of the following in term of $\log_a x$ and $\log_a y$

[Ans : a) $2\log_a x - \frac{1}{2}\log_a y$ b) $\log_a x + 3\log_a y$ c) $\frac{1}{2}\log_a y - \frac{3}{2}\log_a x - 1$]

(a) $\log_a \frac{x^2}{\sqrt{y}}$	(b) $\log_a (xy)^3$	(c) $\log_a \sqrt{\frac{y}{a^2x^3}}$

Exercises 12 : Express each of the following in term of $\log_a x$ and $\log_a y$

[Ans : a) $3\log_a x + 2\log_a y$ b) $3\log_a x - \log_a y$ c) $\frac{3}{2}\log_a x - \log_a y - 2$]

(a) $\log_a x^3 y^2$	(b) $\log_a \frac{x^3}{y}$	(c) $\log_a \sqrt{\frac{x^3}{y^2 a^4}}$

Example 13 : Express each of the following as a single logarithm.

[Ans : a) $\log_a \frac{xy}{a}$ b) $\log_a \frac{x^2\sqrt{y}}{a}$ c) $\log_3 \frac{xy^2}{3}$]

(a) $\log_a x + \log_a y - 1$	(b) $2\log_a x - 1 + \frac{1}{2}\log_a y$	(c) $\log_3 x + 2\log_3 y - 1$

Exercises 13 : Express each of the following as a single logarithm

[Ans : a) $\log_2 x y^2$ (b) $\log_b xyb$ (c)) $\log_4 \frac{\sqrt{x}}{16 y^3}$]

(a) $\log_2 x + \log_2 y^2$	(b) $\log_b x + \log_b y + 1$	(c) $\frac{1}{2}\log_4 x - 2 - 3\log_4 y$

Homework : Text Book Exercise 5.2.4 page 79 and skill practice 5.2 page 79

FROM CS

Students will be able to:

3 Understand and use the change of base of logarithms to solve problems

3.1 Find the logarithm of a number by changing the base of the logarithm to a suitable base.

3.2 Solve problems involving the change of base and laws of logarithms.

3.1 Finding the logarithm of a number by changing the base of the logarithm to a suitable base

If a, b, and c are positive number and $a \neq 1$ then

(i) $\log_a b = \frac{\log_c b}{\log_c a}$ (ii) . $\log_a b = \frac{1}{\log_b a}$

Example 14 Find the values of the following

Ans : a) 0.4076 b) -1.152 c) 3.5522

(a) $\log_{30} 4$	(b) $\log_2 0.45$	c) $\log_5 304$

Exercises 14: **Find the values of the following**
Ans : a)1.2925 b)0.8149 c)0.4512

(a)$\log_5 28$	(b)$\log_{200} 75$	(c)$\log_{0.06} 0.32$

Homework : Text Book Exercise 5.3.1 page 80

3.2 Solving problems involving the change of base and laws of logarithms.

Example 15 Given that $\log_5 x = p$, express each of the following in term of p [Ans i)p/2 ii)1/p iii) 2/p + 3]

i) $\log_{25} x$	ii)$\log_x 5$	iii) $\log_x 25x^3$

Exercises 15 Given that $\log_2 5 = t$ express each of the following in term of t [Ans i)3/t ii) 1/t iii) 2/t –1iv) 1- 1/t

i) i) $\log_5 8$	ii)$\log_5 2$	iii)$\log_5 0.8$	iv)$\log_5 2.5$

Example 16 [Ans 2r/s , 4/r+2s]
If $\log_3 x = r$ and $\log_3 y = s$, express each of the following in terms of r and s .

a) $\log_y x^2$ b) $\log_{xy^2} 81$

Exercises 16 [Ans (a) 2m + n)/2 b) (m+2n)/(2m+n)
If $\log_2 x = m$ and $\log_2 y = n$, express each of the following in terms of m and n

a) $\log_4 x^2 y$ b) $\log_{x^2 y} xy^2$

Homework : Text Book Exercise 5.3.2 page 81 skill practice 5.3 page 81

FROM CS

Students will be able to:

4. Solve equations involving indices and logarithms

4.1 Solve equations involving indices.

4.2 Solve equations involving logarithms.

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4.1 Solving equations involving indices

Comparison of indices or bases

If $a^x = a^y$ then $x = y$ or $a^x = b^x$ then $a = b$

Example 17

Solve the following equations : [Ans a) x = ¾ b) 5/3 c) x = 1 , x = 3

a) $16^x = 8$	b) $9^x \bullet 3^{x-1} = 81$	c) $2^{x^2+3} - 4^{2x} = 0$

Exercises 17: Solve the following equations: J: a)2/3 b) – 1 c) 7/5

a)$9^{x-2} = 27^{x-1}$	b) $8^x = 4$	c) $2^{3x} - 4^{x-1} = 32$

Example 18

Solve the following equations : [Ans (a) 0.8614 b) 0.8174 c) 0.1123]

a) $5^x = 4$	b) $0.8^{2x-1} = 25^{x+1}$	c) $2^{2x} \bullet 5^{x+1} = 7$

Exercises 18: Solve the following equations: Ans : (a) x = 0.7713 b) x = 21.67 (c) 8.837

a) $3^{x+1} = 7$	b) $2^x \bullet 3^x = 9^{x-4}$	c) $2^x \bullet 3^x = 5^{x+1}$

Example 19 Solve the following simultaneous equations : [Ans x = -1 , y = 1 b) x = 2 y = 1]

a) $2^{x} \bullet 4^{2y} - 8$ and $\frac{3^{x}}{9^{y}} = \frac{1}{27}$	(b) $2(4^{x}) = 32^{y}$ and $\frac{27^{x}}{9^{y}} = 81$

Exercises 19: Solve the following simultaneous equations : Ans : (a) x = 3 , y = 2 b) x = 3, y = 2

a) $2^{x} \bullet 4^{y-x} = 32$ and $\frac{3^{2x}}{3^{5y}} = \frac{1}{81}$	b) $3^{x} \cdot 9^{y-1} = 243$ and $\frac{2^{3x}}{4^{y}} = 32$

Homework : Text Book Exercise 5.4.1 page 83

4.2 Solve equations involving logarithms

For two logarithms of the same base,
if $\log_a m = \log_a n$ then $m = n$

Example 20 : Solve the following equations: Ans a) x = 14/3 b) x = 2 c) x = 3

a) $\log_5 (3x - 5) = 2\log_5 6 - \log_5 4$	b) $\log_x 8 = 5 - \log_x 4$	c) $\log_4 (x + 6) = \log_2 3$

Exercises 20 : Solve the following equations: Ans a) :a) x = 11 b) x = 2 c) x = 3 / 2

a) $\log_3 2 + \log_3 (x + 5) = \log_3 (3x - 1)$	b) b) $\log_4 (x + 6) = \log_2 3$	c) $3 + \log_x 4 = 5 \log_x 2$

Homework : Text Book Exercise 5.4.2 page 83 Skill Practice 5.4 Pg 84

SPM Questions

SPM 2003	SPM 2004	SPM 2005
Given that $\log_2 T - \log_4 V = 3$, express T in terms of V	Solve the equation $32^{4x} = 4^{8x+6}$	Solve the equation $2^{x+4} - 2^{x+3} = 1$
		Solve the equation $\log_3 4x - \log_3 (2x-1) = 1$
Solve the equations $4^{2x-1} = 7^x$	Given that $\log_5 2 = m$ and $\log_5 7 = p$ express $\log_5 4.9$ in terms of m and p	
		Given that $\log m_2 = p$ and $\log m_3 = r$, express I $\log_m \left(\frac{27m}{4} \right)$