

module module

TEACHING & LEARNING

ADDITIONAL MATHEMATICS

FORM 4

QUADRATIC EQUATIONS

NAME:.....

FORM :.....

Date of getting the module:

Date of completing the module

Marks of the Topical Test :

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For Internal Circulations Only

LEARNING

OBJECTIVE

In this subtopic you will learn to :

1. Understand the concept of quadratic equation and its roots
 - 1.1 Recognise a quadratic equation and express it in general form.
 - 1.2 Determine whether a given value is the root of a quadratic equation by
 - a) substitution;
 - b) inspection.
 - 1.3 Determine roots of quadratic equations by trial and improvement method

1. Understand the concept of quadratic equation and its roots.

Note : The general form of a quadratic equation is $ax^2 + bx + c = 0$. where a , b , and c are constants and $a \neq 0$. The highest power of the unknown, (x) , is 2

1. State whether each of the following equations is a quadratic equation or not.

Equations	yes or no	Give your reason
1. $2x + 4 = 0$	<i>no</i>	<i>The highest power of x is one .It's a linear equation</i>
2. $x^2 + 4 = 0$		
3. $3x^2 + 2x + 3 = 0$		
4. $5x = 2 - 3x$		
5. $y(2 - 3y) = 7$		
6. $p(3p - 2) = 4 + 2p$		
7. $\frac{1}{x^2} - 2x = 0$		
8. $(x - 7)^2 = 6$		
9. $5xy + 6 = 0$		
10. $x^3 + 2x = 4$		
11. $(n + 2)(n - 3) = 5$		
12. $\frac{2}{x} + 4x^2 - 3 = 0$		

1.1 Recognise a quadratic equation and express it in general form.

1.1 Rewrite the following quadratic equations in general form and find the value of a , b and c .

Example 1	Example 2	Example 3
$x^2 - 2x = 3$ $x^2 - 2x - 3 = 0$ $a = 1, b = -2, c = -3$	$(3x + 1)(x - 3) = 4$	$x^2 + 1 = \frac{10}{3}x$

a) $2x^2 = 3x - 4$	a) $x(2x - 1) = x + 5$	a) $\frac{2}{3}(x^2 - 3x) = x$
b) $n(2n - 1) = 3n$	b) $(3x + 2)^2 = 8$	b) $2x^2 - 1 = \frac{1}{4}x$
c) $x^2 - 1 = 5x - 3p$	c) $3x^2 - 5 = 4x(1 - x)$	c) $2x^2 + \frac{x}{3} = 5$
d) $3x^2 + 4x = 1 - 2p$	d) $x^2 + 2mx = 3x - 5$	d) $2x^2 + 5x = p(1 - 2x)$
e) $4x^2 = x + 5k - 3$	e) $5x^2 - x = 3kx - 4$	e) $px^2 + 4x + 3q = 1 + 2x$

Homework : Text Book page 26 Exercise 2. 1.1

1.2 Determine whether a given value is the root of a quadratic equation by

- a) substitution;
- b) inspection.

Note : The root of a quadratic equation is the value of the unknown in the equation which satisfies the equation . If a value is given, it can be determined whether it is a root by substitution or inspection.

1 Determine whether the x value given are the roots of the following quadratic equations .

Quadratic equations	x value	Root/ No
1. $x^2 - 2x - 3 = 0$	$x = 1, \quad 1^2 - 2(1) - 3 = -4$	No
	$x = -1, \quad (-1)^2 - 2(-1) - 3 = 0$	Yes
	$x = 3, \quad 3^2 - 2(3) - 3 = 0$	
	$x = -3, \quad (-3)^2 - 2(-3) - 3 = 12$	
2. $3x^2 - 5x - 12 = 0$	$x = 1,$	
	$x = 3,$	
	$x = -3,$	
	$x = -\frac{4}{3},$	
3. $(2x + 1)(x - 4) = 0$	$x = 1,$	
	$x = 4,$	
	$x = -3,$	
	$x = -\frac{1}{2},$	

Homework : Text Book page 28 Exercise 2. 1.2

1.3 Determine roots of quadratic equations by trial and improvement method.

Trial and improvement method is a primitive method of repeated substitution of integers into a function or polynomials to find solutions. (Synonymous to trial and error method)

Example 1.3

Find the roots of the quadratic equation $x^2 - 5x + 6 = 0$ by using trial and improvement method

Solution:

Trial	x	$x^2 - 5x + 6$	Trial	x	$x^2 - 5x + 6$
First	-3		Fourth	2	
Second	3		Fifth	-2	
Third	1		Sixth	6	

Homework : Text Book page 29 Exercise 2. 1.3 and Skill Practice 2.1 page 29

LEARNING OBJECTIVE

What we should learnt in this subtopic are :

2. Understand the concept of quadratic equations
 - 2.1 Determine the roots of a quadratic equation by
 - a) factorisation;
 - b) completing the square
 - c) using the formula.
 - 2.2 Form a quadratic equation from given roots

- 2.1 Determine the roots of a quadratic equation by
 a) factorisation; b) completing the square c) using the formula d) using calculator

factorisation	completing the square	Using a formula
$ax^2 + bx + c = 0$ can be factorised completely by converting on the left hand side as a product of two linear factor. $(x-a)(x-b) = 0$ $x-a = 0$ or $x-b = 0$ $x = a$ or $x = b$	completing the square is converting an expression or equation into the "perfect square" form. Converting from general form to perfect square form i.e. $y = ax^2 + bx + c$ to $y = (x+a)^2 + d$ Eg. $2x^2 - 8x + 5 = 2(x-2)^2 - 3$ It used to find the roots of quadratic equations when the quadratic equations , cannot factorise.	Quadratic Equations can also be solved by using the formula as follows : $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ where a ,b, c are constants and related to $ax^2 + bx + c = 0$

Example 2.1

a) Solve the equation $(2x+1)(3x-2) = 3$ by factorisation .	b) Solve the equation $x^2 + 5x - 2 = 0$ by completing the square .	c) Solve the equation $x^2 - 10x - 3 = 0$ by using a formula .
d) Solve the equation $4x^2 + 12x + 9 = 0$ by factorisation	Solve the equation $2x^2 - 4x - 3 = 0$ by completing the square .	Solve the equation $3x(x-2) - x(x-11) = 3$ by using a formula .

Exercise 2.1

1) Solve the following quadratic equation by factorisation.

a) . $12x^2 + 7x = 10$

Ans: $\frac{-5}{4}, \frac{2}{3}$

b) . $9y^2 = 12y - 4$

Ans: $\frac{2}{3}$

2. Solve the following quadratic equation by completing the square. Give your answers correct to 4 significant figure

a) . $2y^2 + 6y + 3 = 0$

Ans : -2.366; -0.634

b) . $(x-1)(x-5) = 25$

Ans: -2.385; 8.385

3. Solve the following quadratic equation by using formula . Give your answers correct to 4 significant figure

A) $5x^2 = 2x + 2$

Ans:-0.463;0.863

b) $5x^2 = 2x + 2$

Ans: $\frac{1}{2}, -3$

Note : Solve all quadratic equations above using calculator

Homework : Text Book page 32 Exercise 2. 2.1

2.2 Form a quadratic equation from given roots

1. If a and b are the roots of a quadratic equation then
 $x = a$ or $x = b$
 $x - a = 0$ or $x - b = 0$
 $(x - a)(x - b) = 0$, hence $x^2 - (a + b)x + ab = 0$
 Therefore, the quadratic equation with roots P and q is
 $x^2 - (a+b)x + ab = 0$

2. The Step of forming a quadratic equation from given roots are

- i. Find the sum of the roots
- ii. Find the product of the roots
- iii. Form a quadratic equation by writing in a following form

$x^2 - (\text{sum of the roots})x + \text{product of the roots} = 0$

Example 2.2(i) Form the quadratic equation whose roots are shown below

a). 3 and -5	b). 4 and $\frac{2}{3}$	c) 3α and 2α
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Exercise 2.2

a). $-\frac{1}{7}$	b). $\frac{1}{2}$ and $-2\frac{1}{3}$	c) $4r$ and $5r$
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Example 2.2(ii) State the sum and product of the roots of the following quadratic equations.

a). $x^2 - 9x - 4 = 0$	b). $3x^2 + 5x + 4 = 0$	c) $x(x - 1) = 2(1 - x)$
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Exercise 2.2(ii) State the sum and product of the roots of the following quadratic equations.

a). $x^2 + 4x + 5 = 0$	b). $2x^2 - 6x + 3 = 0$	c)). $2x^2 + (t+2)x + t^2 = 0$
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Example 2.2(ii)

If α and β are the roots of the equation $2x^2 + 5x - 6 = 0$, Form the equations whose roots are

a) $\alpha + 1, \beta + 1$	b) $\frac{2}{\alpha}, \frac{2}{\beta}$
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Exercise 2.2(ii)

If α and β are the roots of the equation $3x^2 - 2x + 4 = 0$, Form the equations whose roots are

a) $2\alpha + 1, 2\beta + 1$	b) $\frac{1}{\alpha}, \frac{1}{\beta}$
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Example 2.2(iii)

1. If One root of the equations $27x^2 + kx - 8 = 0$ is square the other .Find the value of k

Exercise 2.2(iii)

2. If one root of the equations $2x^2 + x - c = 0$ is two times the other, find the value of c

3. Find q if the equation $3x^2 - 4x + q = 0$ has equal roots

4. If the roots of the equation $x^2 + px + 7 = 0$ are denoted by α and β , and $\alpha^2 + \beta^2 = 22$. find the possible values of p (Camb)

5. Given that α and β are the roots of the equation $x^2 - 2x + 3 = 0$, Find a quadratic equation whose roots are $\alpha^2 + \beta^2$ and $2\alpha\beta$ (Camb)

6. . Given that α and β are the roots of the equation $2x^2 - 3x + 4 = 0$, Write down the value of $\alpha + \beta$ and $2\alpha\beta$. Find an equation whose roots are $\alpha + \frac{1}{\alpha}$ and $\beta + \frac{1}{\beta}$ (Camb)

<p>3. Understand and use the conditions for quadratic equations to have</p> <p>a) two different roots;</p> <p>b) two equal roots;</p> <p>c) no roots / no real roots</p> <p>3.1 Determine types of roots of quadratic equations from the value of $b^2 - 4ac$.</p> <p>3.2 Solve problems involving $b^2 - 4ac$ in quadratic equations to:</p> <p>a) find an unknown value;</p> <p>b) derive a relation.</p>

3.1 Determine types of roots of quadratic equations from the value of $b^2 - 4ac$.

For the quadratic equation $ax^2 + bx + c = 0$, the discriminant of the equation is $b^2 - 4ac$

Types of roots of quadratic equations from the value of $b^2 - 4ac$

- (i) $b^2 - 4ac > 0$ Two different roots (the roots are distinct)
- (ii) $b^2 - 4ac = 0$... Two same roots
- (iii) $b^2 - 4ac < 0$ No real roots

example 3.1 Determine the type of the roots of the following quadratic equations

$ax^2 + bx + c = 0$	a	b	c	$b^2 - 4ac$	Type of roots
1. $x^2 + 5x + 6 = 0$	1	5	6	1	Two different roots
2. $x^2 + 6x + 9 = 0$					
3. $4x^2 - 4x + 1 = 0$					
4. $2x^2 - 4x - 5 = 0$					
5. $2x^2 - 5x + 4 = 0$					

Homework : Text Book page 36 Exercise 2. 3.1.

3.2 Solve problems involving $b^2 - 4ac$ in quadratic equations to:

- a) find an unknown value;
- b) derive a relation.

The value $b^2 - 4ac$ can be used to find the unknown value of coefficients or to derive a relation which involves unknown in the quadratic equations

Example 3.2

a) Find k if $x^2 + 8x + k = 0$ has equal roots	b) Find p if $3x^2 + 2x + 3p = 0$, has two different roots

Exercise 3.2

<p>a) Find The range of value of p if $3x^2 - 1 = 6x - 2p$ has two distinct roots</p> <p style="text-align: right;">[p < 2]</p>	<p>Find The range of value of p if $x^2 + 2x + 9 = p(2x - p)$ has two different roots</p> <p style="text-align: right;">[P > - 4]</p>
<p>c) .The quadratic equation $3x^2 + 2x + h = 0$ has equal roots . Find the value of h</p> <p style="text-align: right;">[h = $\frac{1}{3}$]</p>	<p>d) Show that the roots of the equations $6x - 6 - 2px^2 = x^2$ are complex if $p > \frac{1}{4}$</p>
<p>e) Given that $x^2 + (p- 2)x + 10 - p = 0$ has two equal roots, find the values of p .</p> <p style="text-align: right;">[P = ± 6]</p>	<p>Find the range of values of h for the quadratic equation $2x^2 + 3x + 4p = 1$ which has no roots</p> <p style="text-align: right;">[p > $\frac{17}{32}$]</p>

SPM QUESTIONS

<p>SPM 2003 [2.591 or - 0.2573] Solve the quadratic equation $2x(x - 4) = (1 - x)(x + 2)$. Give your answer correct to four significant figure [2 marks]</p>	<p>SPM 2004 Form the quadratic equations which has the roots – 3 and $\frac{1}{2}$. Give your answer in the form $ax^2 + bx + c = 0$, where a, b and c are constants [2 marks]</p>
<p>SPM 2004 Form the quadratic equations which has the roots – 3 and $\frac{1}{2}$. Give your answer in the form $ax^2 + bx + c = 0$, where a, b and c are constants</p>	<p>SPM 2003 [$p < -3$ or $p > 5$] The quadratic equation $x(x + 1) = px - 4$ has two distinct roots. Find the range of values of p</p>
<p>SPM 2005 Solve the quadratic equation $x(2x - 5) = 2x - 1$ Give your answer correct to three decimal place</p>	<p>SPM 2006 A quadratic equation $x^2 + px + 9 = 2x$ has two equal roots. Find the possible values of p</p>