

module

module

TEACHING & LEARNING

ADDITIONAL MATHEMATICS

FORM 4

QUADRATIC FUNCTIONS

CHAPTER 3

NAME:.....

FORM :.....

Date received :

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Marks of the Topical Test :

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For Internal Circulations Only

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1. Understand the concept of quadratic functions and their graphs.
 - 1.1 Recognise quadratic functions.
 - 1.2 Plot quadratic function graphs
 - a) based on given tabulated values;
 - b) by tabulating values based on given functions.
 - 1.3 Recognise shapes of graphs of quadratic functions.
 - 1.4 Relate the position of quadratic function graphs with types of roots for $f(x) = 0$.

1. Understand the concept of quadratic functions and their graphs.

1.1 Recognise quadratic functions.

The general form of quadratic functions is $f(x) = ax^2 + bx + c$. The highest power of variable x is 2

Example 1.1

Determine whether the given functions is a quadratic functions . If so Express it in the general form

a) $f(x) = -3x + 2x^2 - 5$	b) $f(x) = (2x - 1)(3x + 1)$	c) $f(x) = (x - 3)^2 - 8$
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Exercise 1.1

Determine whether the given functions is a quadratic functions . If so Express it in the general form

a) $f(x) = -5x + 3x^2 - 5$	b) $f(x) = (3x - 4)(2x + 1)$	c) $f(x) = (x - 2)^2 + 8$
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Home work : Text book page 44 Exercise 3.1.1

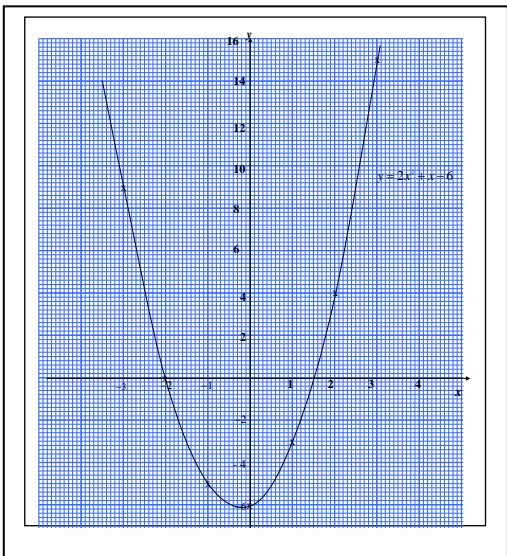
1.2 Plot quadratic function graphs

- a) based on given tabulated values;
- b) by tabulating values based on given functions.

Example 1.2

a) The table below shows the tabulated values of a quadratic function $f(x)$. By suitable scale, plot the graph of the function $f(x)$

x	-3	-2	-1	0	1	2	3
y	9	0	-5	-6	-3	4	15

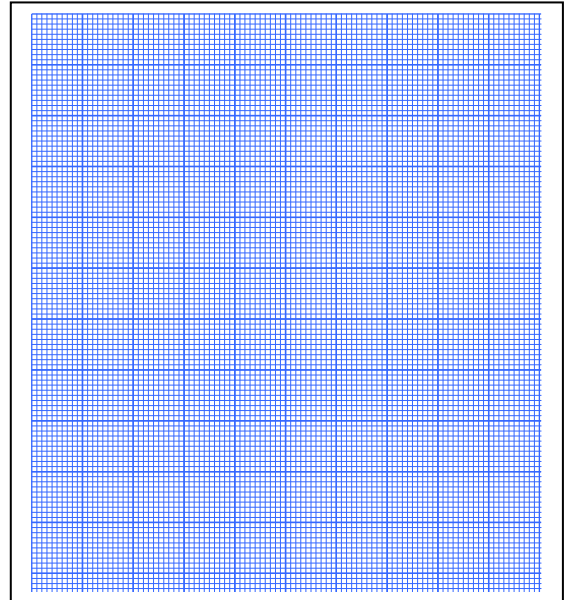


b) Given function $f(x) = 4 - 3x - x^2$.

Tabulate values for x and $f(x)$

$-3 \leq x \leq 3$ Then by using a suitable scale.

Plot the graph of $f(x) = 4 - 3x - x^2$.



x	-3	-2	-1	0	1	2	3
$f(x)$							

Home work : Text book page 45 Exercise 3.1.2

1.3 Recognise shapes of graphs of quadratic functions.

Note : Maximum and minimum values of $ax^2 + bx + c$

If $a > 0$ the function has a minimum value and the shape of a graph is



If $a < 0$ the function has a maximum value and the shape of a graph is



Example 1.3

Identify the shape of the graph of the following quadratic function

a) $f(x) = 6 + 4x - 2x^2$	b) $f(x) = x^2 - 4x + 3$	c) $f(x) = (4 - x)(2x - 5)$
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Home work : Text book page 46 Exercise 3.1.3

1.4 Relate the position of quadratic function graphs with types of roots for $f(x) = 0$.

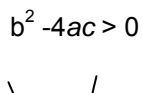
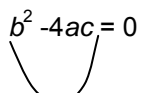
Note : Quadratic functions

Position of quadratic function

$$f(x) = ax^2 + bx + c$$

(i) $a > 0$

$$b^2 - 4ac < 0$$



(ii) $a < 0$

$$b^2 - 4ac < 0$$



$$b^2 - 4ac = 0$$



$$b^2 - 4ac > 0$$



$b^2 - 4ac$	$a > 0$	$a < 0$

Example 1.4(i) Determine the type of the roots of the following quadratic equations for $f(x) = 0$ and sketch the position of the graph $f(x)$ relative to the x – axis

a) $f(x) = 6 - x - x^2$	b) $f(x) = 4x^2 + 4x + 1$	c) $f(x) = 5x^2 - x + 1$
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Exercise 1.4 (i) Determine the type of the roots of the following quadratic equations for $f(x) = 0$ and sketch the position of the graph $f(x)$ relative to the x – axis

a) $f(x) = x^2 - 3x + 5$	b) $f(x) = 2 + 3x - 2x^2$	d) $f(x) = x^2 - 4x + 4$
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Example 1.4 (ii)

<p>a) Find the value of p if the graph of the quadratic function $f(x) = x^2 - 2px + 4p + 5$ touches the x axis at one point. [$p = 5$ or $p = -1$]</p>	<p>b) Find the range of values of k for which the quadratic function $g(x) = 2x^2 - 12x + 3 - k$ has two x – intercept [$k > -15$]</p>	<p>C) Find the range of values of m which the quadratic function $f(x) = x^2 - 8x + 5m - 4$ does not intersect the x-axis . [$m > 4$]</p>
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Exercise 1.4(ii)

<p>a) Find the value of p if the graph of the quadratic $f(x) = px^2 + (2p + 6)x + 5p - 3$ touches the x axis at one point. [$p = -3/4, 3$]</p>	<p>b)) Find the range of values of k for which the quadratic function $g(x) = (p+5)x^2 - 8x + 8$ has two x – intercept [$p < -3$]</p>	<p>a) Show that the graph of quadratic function $f(x) = tx^2 + (3 - 2t)x - 5 + t$ intercept the x-axis in two distinct point if $t > \frac{9}{8}$</p>
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Home work : Text book page 49 Exercise 3.1.4 and Skill Practice 3.1 page 50.

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<p>2. Find the maximum and minimum values of quadratic functions.</p> <p>2.1 Determine the maximum or minimum value of a quadratic function by completing the square.</p>

Finding the maximum and minimum values of quadratic functions.

Note : A maximum or a minimum value of a quadratic function can be expressed in the form $a(x + p)^2 + q$, where a, p , and q are constants by completing the square .

This can be done as follows

$f(x) = ax^2 + bx + c$ $=$

2.1 Determine the maximum or minimum value of a quadratic function by completing the square.

Example 2.1 (i)

In each of the following, state the maximum or minimum value of $f(x)$ and the corresponding value of x

<p>a) $f(x) = 3(x + 5)^2 + 6$</p>	<p>b) $g(x) = 8 - 2(x-9)^2$</p>	<p>c) $h(x) = 16 - x^2$</p>
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Class exercises

In each of the following, state the maximum or minimum value of $f(x)$ and the corresponding value of x

a) $f(x) = 15 - 3(x + 4)^2$	b) $h(x) = (x - 5)^2 + 8$	c) $m(x) = 4 - x^2$
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Example 2.1(ii)

Find the maximum or minimum values of the following quadratic function and the values of x when these occur

a) $f(x) = 2x^2 - 6x + 7$	b) $g(x) = 4 + 12x - 3x^2$
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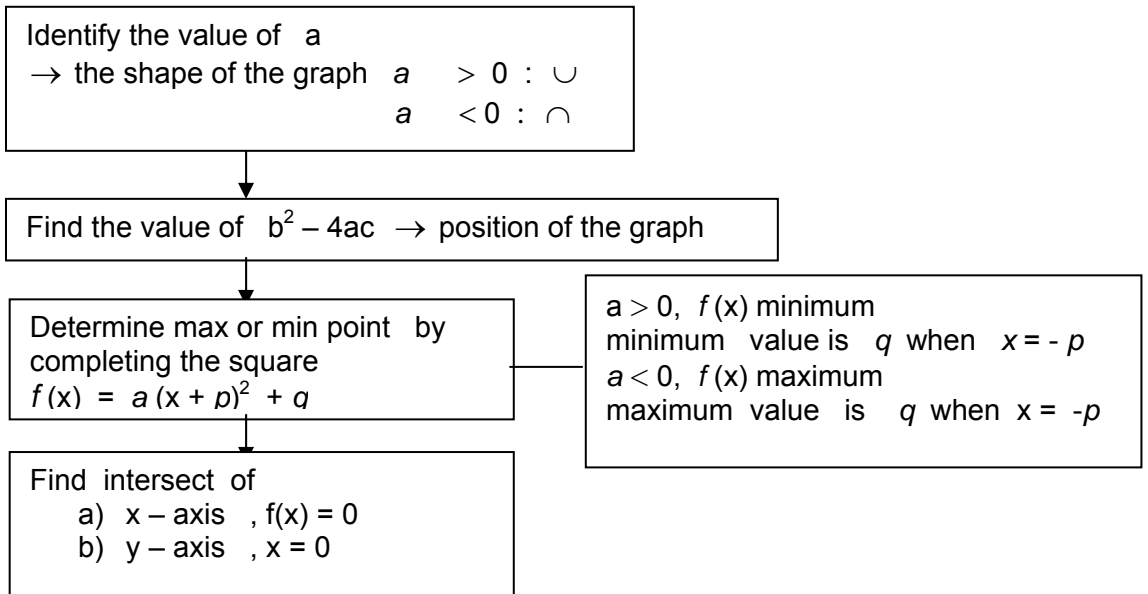
Class exercises

Find the maximum or minimum values of the following quadratic function and the values of x when these occur

a) $h(x) = 2x - 1 - 3x^2$ [max = $-2/3$, $x = 1/3$]	b) $s(x) = 3x^2 - 4x - 2$ [min = $-10/3$, $x = 2/3$]
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3. Sketch graphs of quadratic functions
 3.1 Sketch quadratic function graphs by determining the maximum or minimum point and two other points

A quadratic function can be sketched by the following step :



Example 3.1

a) Sketch the graph of quadratic function $y = 3x^2 + 5x + 2$. State the axis of symmetry and also the coordinate of the maximum or the minimum point.

b) Find the maximum value of the quadratic function $f(x) = 2x - 3 - x^2$, then state the axis of symmetry and sketch the graph.

Class exercises

a) Find the maximum value of the quadratic function $y = 2x^2 + 2x + 2$, then state the axis of symmetry and sketch the graph.

b) Sketch the graph of quadratic function $f(x) = 2 + 3x - 2x^2$. State the axis of symmetry and also the coordinate of the maximum or the minimum point.

4. Understand and use the concept of quadratic inequalities.
 4.1 Determine the ranges of values of x that satisfies quadratic inequalities

QUADRATIC INEQUALITIES.

The range of values of x which satisfies this inequality can be found from this step

- i. Find the intersects of x - axis
- ii. Determine the shape of the graph
- iii. Sketch the graph
- iv. From the graph the range of values of x which satisfies this inequality can be found

Alternative method of finding the range of values of x to draw a number line. Expressions involving more than two factors can be similarly treated .

Example 4.1

Find the range of value of x if $2x^2 < 5x + 3$

Method 1: Graph

Method 2: Number line

2. Find the range of value of x for the following quadratic inequalities.

a) $\frac{(x-4)(x+2)}{x+3} \geq 0$

b) $(2x+1)(x+3) > 7$

c) Find the range of values of p which the quadratic equations $3x^2 - px + 2p = 0$ does not has a real roots [$0 < p < 24$]

d) Given that $y = tx^2 + 8x + 10 - t$, Find the range of values of t where y always positive . [$2 < t < 8$]

Exercise 4.1

b) Find the smallest of k with the condition that the equation $kx^2 + (2k - 15)x + k = 0$ does not have a real root. [$k = 4$]

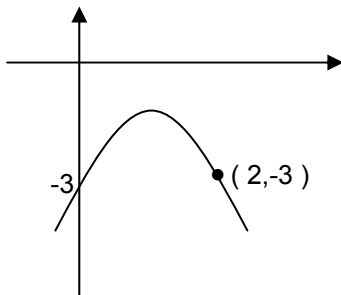
c) Find the range of p if $y = 2x - 2p$ does not intersect at the curve $x^2 + 2y^2 = 8$. [$p < -5, p > 5$]

Home work : Text book page 56 Skill Practice 3.4
Review Exercise page 40

SPM Questions

2004

Find the range of values of x for which $x(x-4) \leq 12$. Diagram 2 shows the graph of function $y = -(x-k)^2 - 2$ where k is a constant



Find

- the value of k
- the equations of the axis of symmetry
- the coordinate of the maximum point

SPM 2003 [$p < -3$ or $p > 5$]

The quadratic equation $x(x+1) = px - 4$ has two distinct roots. Find the range of values of p

The function $f(x) = x^2 - 4kx + 5k^2 + l$ has a minimum value of $r^2 + 2k$, where r and k are constants

- By using the method of completing the square show that $r = k-1$
- Hence or otherwise, find the values of k and r if the graph of function is symmetrical about $x = r^2 - 1$

Enrichment 1 – Quadratic Functions

- Find ranges of values of x which satisfies $3 - 2x - x^2 > 0$
- Find ranges of values of x so that $6 + x > 2x^2$
- Find the ranges of values of m so that $x^2 + 6x + m > 0$ for all values of x .
- If $f(x) = 7 - ax - x^2 = 16 - (x + b)^2$ for all values of x and both a and b is positives, find:
 - The values of a and b
 - The turning point for graph $f(x)$.

(c) The ranges of values of x which satisfies $f(x) \geq 0$

- Find the ranges of values of x that satisfy $-1 \leq x^2 - 4x + 2 \leq 7$
- Determine the ranges of values of x that satisfy each of the following inequalities:
 - $(x + 5)(x - 1) > 0$
 - $2x^2 + 3x > 2$

7. Express each of the following functions in the form of $f(x) = (x + p)^2 + q$, where p and q are constants. Hence, sketch the graph for each of the functions:

- (a) $f(x) = x^2 - 2x - 3$
 (b) $g(x) = 2x^2 - x - 3$
 (c) $h(x) = 2 - x - x^2$
 (d) $f(x) = 6 - 5x - 2x^2$

8. The minimum value of $f(x) = x^2 + ax + b$ is 6 when $x = 2$. Find the values of a and b .

9. A straight line $x + 2y = m$ intersects a curve of $x(x + y) + 9 = 0$ at two different points. Find the ranges of values of m .

10. Sketch the graph of $f(x) = 6 + x - x^2$ for the domain $-3 < x < 4$ and determine the symmetrical axis. Hence, find the corresponding ranges for the graph.

11. Find the ranges of values of x which satisfy each of the following quadratic inequalities :

- (a) $(x - 1)(x + 3) > 0$
 (b) $(2x + 1)(x + 3) \geq 7$
 (c) $6x^2 - 11x < 7$
 (d) $x^2 - 4x - 12 \leq 0$
 (e) $3x + 2 \geq 2x^2$
 (f) $7x - 3 - 2x^2 > 0$

12. Determine the maximum value for $f(x) = 5 - (x + 2)^2$ and state the corresponding value of x . Then sketch the graph.

13. Express the function $f(x) = 9 + 2x(x - 3)$ in the form of $f(x) = (x + p)^2 + q$. Then find the maximum or minimum value for the function of $f(x)$ and the corresponding value of x

Enrichment 2 – Quadratics Functions

1. The function $f(x) = x^2 + ax + b$ have a minimum value of $\frac{-7}{2}$ when $x = \frac{-3}{2}$. Find the values of a and b . Then, sketch the graph for $f(x)$.
2. Find the ranges of values of m if the quadratic function $f(x) = x^2 - 3mx + 3m + 8$ does not intersect the x -axis.
3. Given the function of $f(x) = 6x - 1 - 3x^2$.
- (a) Express the quadratic function in the form of $f(x) = k + m(x + n)^2$, where k , m , and n are constants. Determine whether the function has minimum or maximum values. Hence, state the value
- (b) Sketch the graph for function $f(x)$
- (c) Find the ranges of values of p if the equation $6x - 4 - 3x^2 = p$ has two different roots.
4. (a) Find the ranges of values of x that satisfy $5x \leq x^2$
 (b) Find the ranges of values of p if $x^2 - (p + 1)x + 1 - p^2 = 0$ has no real roots.
5. (a) Given that $3x + 2y - 1 = 0$. Find the ranges of values of x if $y < 5$.

(b) Find the ranges of values of n if $2n^2 + n \geq 1$.

6. $f(x) = 0$ is a quadratic equation which have -3 and p as its roots.
- (a) Write $f(x)$ in the form of $ax^2 + bx + c$.
- (b) The curve of $y = kf(x)$ intersects the y -axis at the point $(0, 60)$. Given $p = 5$, determine:
- (i) the value of k
- (ii) the coordinates of the maximum point of the curve without using differentiation method or drawing graph method.

7. Find the range of values of x if $x(x + 1) < 2$

8. (a) Given that $x = \frac{4 - y}{2}$, find the ranges of values of x if $y > 10$

(b) Find the range of values of x if $x^2 - 2x \leq 3$

9.

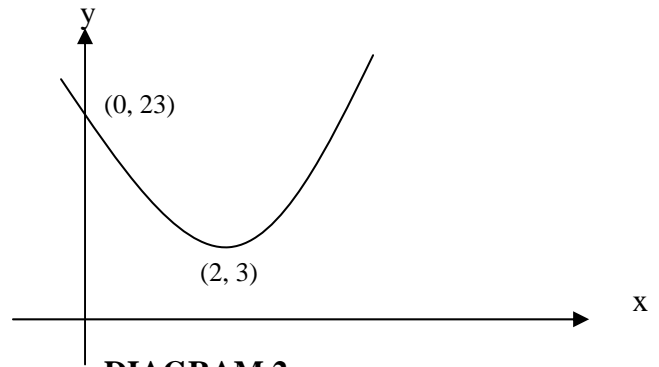


DIAGRAM 2

In the Diagram 2 above, point $(2, 3)$ is a turning point of the graph which has an equation of $y = p(x + h)^2 + k$. find;

- (a) values of p , h and k
- (b) the equation for the curve when the graph is reflected along the x -axis

Enrichment 3 – Quadratic Functions

1. Given that $2x - 3y + 1 = 0$, find the ranges of values of x if $y > 1$.
2. Find the ranges of values of x that satisfy $y - 2x = 1$ and $2xy \leq x + 3$.
3. Given $y = \frac{3 - x}{2}$, find the ranges of values of y if $x < 12$
4. Find the ranges of values of k if the equation $x^2 + (1 + k)x - k^2 + 1 = 0$ has real roots.
5. Determine the ranges of values of k if the quadratic equation $x^2 + (k - 1)x = 1 - k$ has two different roots.
6. Find the ranges of values of p for which

$$(\rho + 1)x^2 - \frac{3}{2}\rho x + \rho - 4 = 0 \text{ has two different roots.}$$

7. Given that $f(x) = 5 + 4x - x^2$, find the ranges of values of x which satisfy the inequalities $5 < f(x) < 8$.

8. Given that $2y(x - 1) < x$, find the possible ranges of values of x , if

(a) $y = \frac{3}{4}$

(b) $y = x$

9. Determine the ranges of values of x which satisfy both inequalities:

$$3 - 2x < 5$$

$$2x^2 + 3x < 2$$

10. Calculate the largest integer of k for which the equation $kx^2 + (2k - 5)x + k = 0$ has real roots.

11. Determine the ranges of values of x that satisfy each of the following inequalities;

(a) $3x + 4 < 5x - 2$

(b) $13x > 12x^2 + 3$

Quadratic Functions – Answers

Enrichment 1	Enrichment 2	Enrichment 3
1. $-3 < x < 1$	1. $f(x) = (x + \frac{a}{2})^2 - \frac{a^2}{4} + b$	1. $x > 1$
2. $-\frac{3}{2} < x < 2$	$a = 3, b = -\frac{5}{4}$	2. $-1 \leq x \leq \frac{3}{4}$
3. $m > 9$	$f(x) = 2(x - \frac{3}{2})^2 + \frac{9}{2}$	3. $y > -\frac{9}{2}$
4. (a) $a = 6, b = 3$	2. $-\frac{4}{3} < m < \frac{8}{3}$	4. $k \leq -1, k \geq \frac{3}{5}$
(b) $(-3, 16)$	3. (a) $f(x) = -3(x - 1)^2 + 2$	5. $k < 1, k > 5$
(c) $-7 \leq x \leq 1$	max value = 2	6. $-\frac{8}{7} < p < 8$
5. $-1 \leq x \leq 1, 3 \leq x \leq 5$	(c) $p < -1$	7. $0 < x < 1,$ $3 < x < 4$
6. (a) $x < -5, x > 1$	4. (a) $x \leq 0, x \geq 5$	8. (a) $x < 3$
(b) $x < -2, x > \frac{1}{2}$	(b) $-1 < p < \frac{3}{5}$	(b) $0 < x < \frac{3}{2}$
7. (a) $f(x) = (x - 1)^2 - 4$	5. (a) $x > -3$	9. $-1 < x < \frac{1}{2}$
(b) $g(x) = 2(x - \frac{1}{4})^2 - \frac{25}{8}$	(b) $n \leq -1, n \geq \frac{1}{2}$	10. $k \leq \frac{5}{4}$
(c) $h(x) = -(x + \frac{1}{2})^2 + \frac{9}{4}$	6. (a) $f(x) = x + (3 - p)x - 3p$	Largest integer of $k = 1$.
(d) $f(x) = -2(x + \frac{5}{4})^2 + \frac{73}{8}$	(b) (i) $k = -4$	12. (a) $x > 3$
8. $a = -4, b = 10$	(ii) $y = -4(x - 1)^2 + 64$	(b) $\frac{1}{3} < x < \frac{3}{4}$
9. $m < -\sqrt{72}, m > \sqrt{72}$	max point = $(1, 64)$	
10. $f(x) = -(x - \frac{1}{2})^2 + \frac{25}{4}$	7. $-2 < x < 1$	
Range: $-6 < f(x) \leq \frac{25}{4}$	8. (a) $x < -3$	
11. (a) $x < -3, x > 1$	(b) $-1 \leq x \leq 3$	
(b) $x \leq -4, x \geq \frac{1}{2}$	9. (a) $h = -2, k = 3, p = 5$	
(c) $-\frac{1}{2} < x < \frac{7}{3}$	(b) $y = -5(x - 2)^2 - 3$	
(d) $-2 \leq x \leq 6$	12. max value = 5, $x = -2$	
(e) $-\frac{1}{2} \leq x \leq 2$	13. $f(x) = 2(x - \frac{3}{2})^2 + \frac{9}{2}$	
(f) $\frac{1}{2} < x < 3$	min value = $\frac{9}{2}, x = \frac{3}{2}$	