

# module module

## TEACHING & LEARNING

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### ADDITIONAL MATHEMATICS FORM 4

# SOLUTION OF TRIANGLES

## CHAPTER 10

NAME:.....

FORM :.....

Date received : .....

Date completed .....

Marks of the Topical Test : .....

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Additional Mathematics Department  
Sek Men Sains Muzaffar Syah Melaka  
For Internal Circulations Only

a)  $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

b)  $a^2 = b^2 + c^2 - 2bc \cos A$

c) Area of triangle =  $\frac{1}{2}ab \sin C$

Students will be able to:

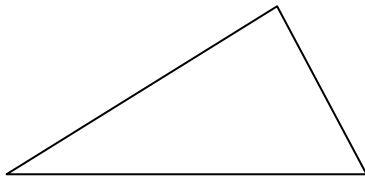
1. Understand and use the concept of sine rule to solve problems.
  - 1.1 Verify sine rule.
  - 1.2 Use sine rule to find unknown sides or angles of a triangle.
  - 1.3 Find the unknown sides and angles of a triangle involving ambiguous case.
  - 1.4 Solve problems involving the sine rule



To solve the triangle means we must find three interior angle and three sides of any triangle

### A Sine Rule

Label the triangle with alphabets  $A, B, C$  and  $a, b, c$ . For any triangle  $ABC$ , whether it is an acute-angled or obtuse-angled triangle, sine Rule state that :

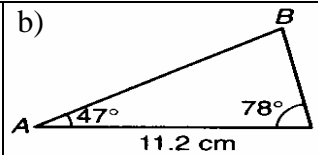
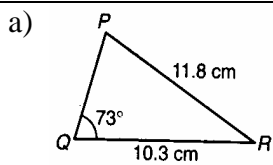


$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

It is correct when the label of the triangle are correct. So label on the triangle is vary importance to verify sine rule.

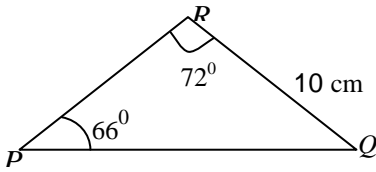
### Homework Text Book Exercise 10.1.1 page 242

Example 1 . Solve each of the following triangle

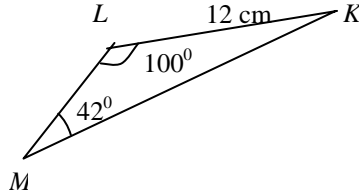


Exercise 1 : Solve the triangles below

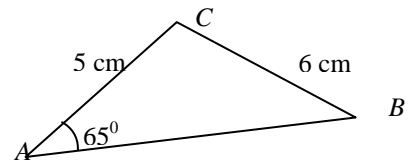
a) Ans  $\angle PQR = 42^\circ$ ,  $PQ = 10.49$  cm,  $PR = 7.38$



2. . Ans  $\angle MKL = 38^\circ$ ,  $MK = 17.66$  cm,  $ML = 11.04$  cm



3. Ans  $\angle ABC = 49.05^\circ$ ,  $\angle ACB = 65.95^\circ$ ,  $AB = 6.05$  cm

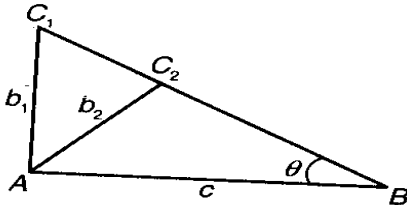


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1.3 Finding the unknown sides and angles of a triangle involving ambiguous case.

Ambiguous case (Ambiguous means having more than one value)

- When two sides and an acute angle which is not an included angle are given with the acute angle opposite the shorter of two given sides, then ambiguity arises.
- Ambiguity means it is possible to form two triangles as shown below



A situation like this will not happen if

- the angle given is an obtuse angle
- the side that is opposite the given angle is longer than the second side.

Example 2 : a) { Answer  $C = 57.7^\circ$  or  $123.3^\circ$  BC 12.8 cm or 23.5cm

a) ABC is a triangle with  $\angle B = 25^\circ$ ,  $AB = 20$  cm and  $AC = 10$  cm. Solve the triangle.

Exercice 2: For each of the following triangle , state if ambiguity arises. Hence, solve the triangles

a)  $\Delta ABC$  :  $\angle B = 30^\circ$ ,  $c = 8\text{cm}$  and  $b = 6\text{cm}$

$\angle C = 41^\circ 49'$ ,  $138^\circ 11'$   $\angle A = 108^\circ 11'$   $110^\circ 49'$   $a = 11.4\text{ cm}$ ,  $2.46\text{ cm}$

c)  $\Delta ABC$  :  $\angle B = 64^\circ 22'$   $a = 13.3\text{ cm}$  and  $b = 12\text{ cm}$

$\angle A = 87^\circ 47'$ ,  $92^\circ 13'$   $\angle C = 27^\circ 51'$   $23^\circ 25'$   $c = 6.22$ ,  $5.29$

b)  $\Delta XYZ$  :  $\angle Y = 69^\circ 10'$ ,  $x = 8.2\text{cm}$  and  $y = 9.5\text{ cm}$

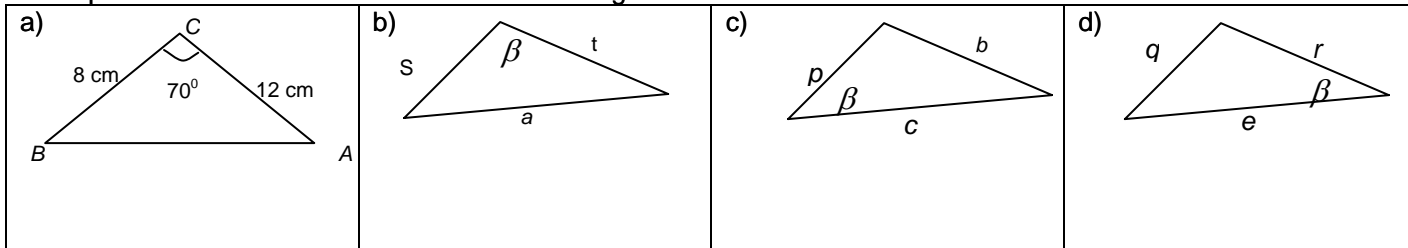
$\angle X = 53^\circ 47'$   $\angle Z = 57^\circ 3'$   $z = 8.53$

d)  $\Delta PQR$  :  $\angle R = 28^\circ$ ,  $p = 8\text{ cm}$  and  $r = 11\text{ cm}$

$\angle P = 19^\circ 58'$   $\angle Q = 132^\circ 2'$   $q = 17.4\text{ cm}$



Example 5: Write the cosine rule for each of triangles below



**Homework Text Book Exercise 10.2.1 page 247**

2.2 Using cosine rule to find unknown sides or angles of a triangle

Example 4

<p>a) Given <math>\triangle ABC</math> such that <math>\angle A = 50.5^\circ</math>, <math>c = 4.5</math> cm and <math>b = 6.5</math> cm. Find the value of <math>a</math> [ Ans <math>a = 5.029</math> cm ]</p>	<p>b) Find all the angle of triangle PQR. Given that <math>p = 4</math> cm, <math>q = 6</math> cm, and <math>r = 8</math> cm. (Ans <math>\angle R = 103^\circ 29'</math>, <math>\angle P = 28^\circ 58'</math>, <math>\angle Q = 46^\circ 33'</math>)</p>
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Exercise 4

<p>a) Given that <math>\triangle ABC</math>, <math>b = 5</math> cm, <math>c = 4</math> cm and <math>\angle A = 66^\circ</math>. Find the value of <math>a</math> [ Answer 4.97 ]</p>	<p>b) Given that <math>\triangle ABC</math>, <math>a = 12.5</math> cm, <math>b = 20.5</math> cm and <math>\angle C = 124^\circ 25'</math>. Find the value of <math>c</math> and the remaining angle of the triangle. [ Answer <math>c = 29.43</math> cm, <math>\angle A = 20^\circ 31'</math>, <math>\angle C = 41^\circ 25'</math> ]</p>
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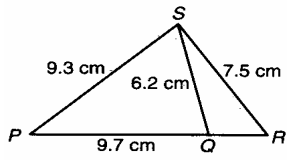
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2.3 Solve problems involving cosine rule

2.4 Solve problems involving sine and cosine rules

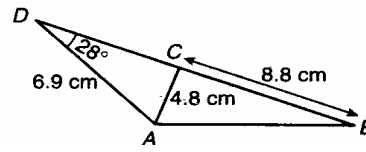
Example 5

From the diagram given, find  $\angle PQS$  and  $\angle RSQ$   
 [ Answer  $67.49^\circ$  and  $17.7^\circ$  ]



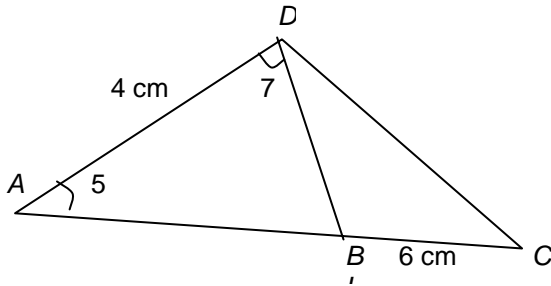
Example 6

From the diagram given, find i)  $\angle ACD$   
 ii) length of  $AB$  [ Answer  $42.4$ ,  $12.76$  cm ]

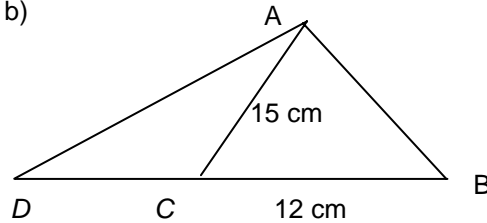


Exercise 6

a) In the diagram below,  $ABC$  is a straight line. Find  
 (a) length of  $BD$  (b) length of  $CD$  [Ans  $3.538$  cm  $8.35$  cm ]



b)



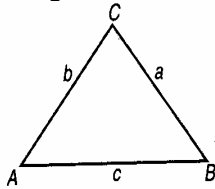
$BCD$  is a straight line in the above diagram. Given that  $BC = 12$  cm,  $AC = 15$  cm,  $\angle CAD = 29^\circ$  and  $\angle ABC = 68^\circ$ .

a) Find  $\angle BAC$ , b) the length of  $CD$   
 [ answer  $47.88^\circ$ ,  $12.64$  cm ]

Students will be able to:

3. Understand and use the formula for areas of triangles to solve problems.
- 3.1 Find the area of triangles using the formula  $\frac{1}{2}ab \sin C$  or its equivalent.
- 3.2 Solve problems involving three-dimensional objects.

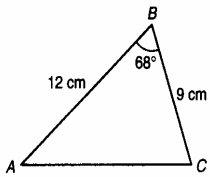
Area of Triangles



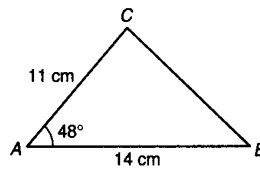
$$\begin{aligned} \text{Area of } \triangle ABC &= \frac{1}{2} ab \sin C \\ &= \frac{1}{2} ac \sin B \\ &= \frac{1}{2} bc \sin A \end{aligned}$$

Find the area of the following triangles

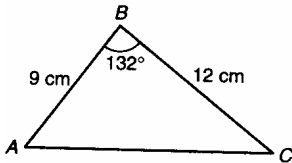
a) Answer  $50.07 \text{ cm}^2$



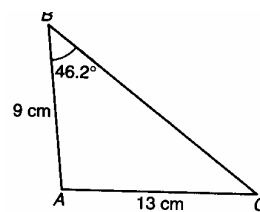
b) Answer [  $57.22 \text{ cm}^2$  ]



c) Answer [  $40.13 \text{ cm}^2$  ]



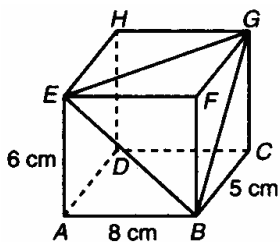
d) Answer [  $56.82 \text{ cm}^2$  ]



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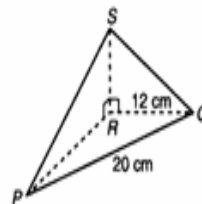
3.2 Solve problems involving three-dimensional objects

a) Calculate  $\angle BGE$  [ Answer  $70.17^\circ$  ]



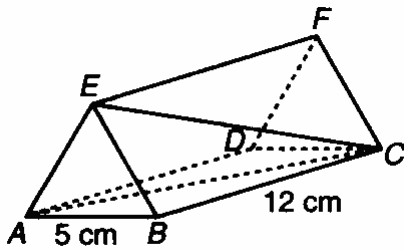
b) Calculate i) Length of PS ii)  $\angle PSQ$

[ Answer  $16.76 \text{ cm}$ ,  $83.41^\circ$  ]



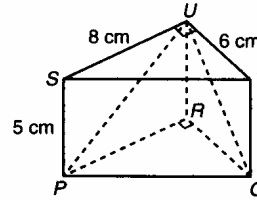
Exercise 7

c) [Answer i)  $25.8^\circ$  ii)  $15.82 \text{ cm}^2$  ]



$ABE$  and  $DCF$  are equilateral triangle  
 $BCFE$  and  $ADFE$  are rectangle.  
 Calculate  
 i)  $\angle ACE$   
 ii) Area of  $\triangle AEB$

d) [  $70.2^\circ$  ii)  $34.66 \text{ cm}^2$  ]



Base on the diagram given on the left  
 Calculate  
 i)  $\angle PUQ$   
 ii) area of  $\triangle PUQ$

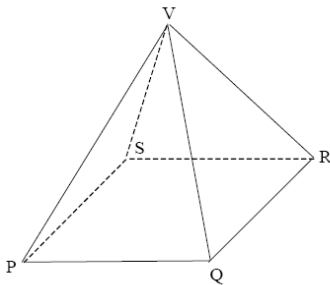


DIAGRAM 6

(b) Diagram 6 shows a right pyramid with a square base. It is given that  $VQ = 10 \text{ cm}$  and  $PQ = 12 \text{ cm}$ .

- (i) Calculate angle  $PRV$ ,
- (ii) If  $T$  is a point on  $VR$  such that  $VT:TR = 3:2$ , calculate the length of  $PT$ .  
[6 marks]

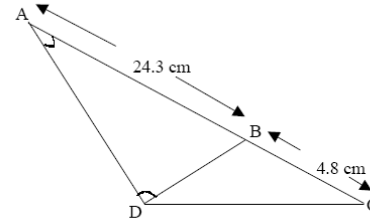


DIAGRAM 5

(a) In Diagram 5,  $ABC$  is a straight line. Calculate the length of  $BD$  and the area of  $\triangle BCD$ .  
[4 marks]

.SPM 2003

A diagram shows a tent VABC in the shape of a pyramid with triangle ABC as the horizontal base. V is the vertex of the tent and the angle between the inclined plane VBC and the base is  $50^\circ$ .

Given that  $VB = VC = 2.2$  m and  $AB = AC = 2.6$  m, calculate

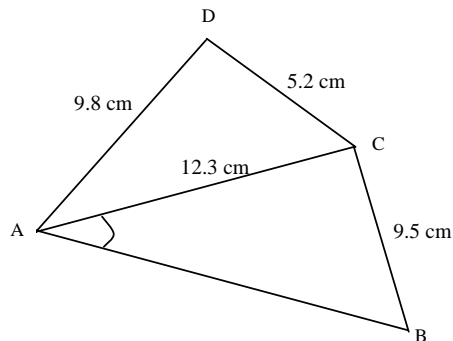
- (a) the length of BC if the area of the base is  $3$  m<sup>2</sup> [3 marks]
- (b) the length of AV if the angle between AV and the base is  $25^\circ$  [3 marks]
- (c) the area of triangle VAB [4 marks]

**Answer:**

- (a) Length BC = 2.700 cm
- (b) Length AV = 3.149 cm
- (c) Area = 2.829 cm<sup>2</sup>

**SPM 2004**

The diagram shows a quadrilateral ABCD such that  $\angle ABC$  is acute angle .

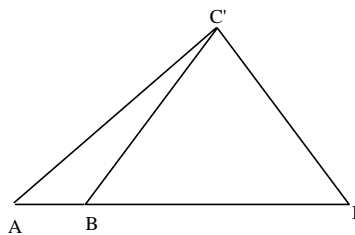


- (a) Calculate
  - (i)  $\angle ABC$
  - (ii)  $\angle ADC$
  - (iii) the area, in cm<sup>2</sup>, of quadrilateral ABCD. [8 marks]
- (b) A triangle A'B'C' has the same measurements as those given for triangle ABC, that is,  $A'C' = 12.3$  cm,  $C'B' = 9.5$  cm and  $\angle B'A'C' = 40.5^\circ$ , but which is different in shape to triangle ABC.
  - (i) Sketch the triangle A'B'C',
  - (ii) State the size of  $\angle A'B'C'$  [2 marks]

**Answer SPM 2004**

- (i)  $\angle ABC = 57.23^\circ$
- (ii)  $\angle ADC = 73.93^\circ$
- (iii) Area of ABCD = 80.96 cm<sup>2</sup>

(b) (i)



- (ii)  $\angle A'B'C' = 180^\circ - 57.23^\circ = 122.77^\circ$