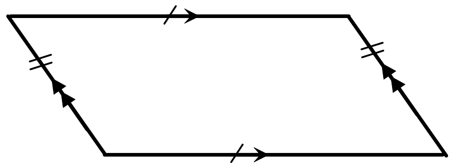
2D Shapes

Quadrilaterals

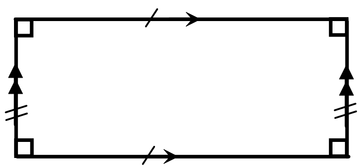
Quadrilaterals are shapes with exactly *four* straight sides. They also have exactly *four* corners. *The interior angles of any quadrilateral add up to 360°.*

Parallelograms



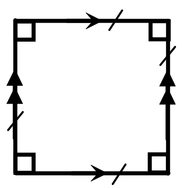
A *parallelogram* is a quadrilateral – it has four sides and four corners. Each side is parallel with the side *opposite* it. Also, the lengths of opposite sides in a parallelogram are the same.

The rectangle



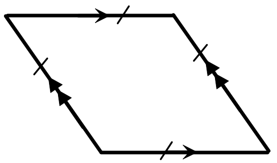
A *rectangle* is a type of parallelogram, with *all* the interior angles exactly 90°.

The square



A *square* is an even more specialised type of parallelogram than the rectangle. A square has all the properties of a rectangle, but on top of these, *all* the sides of a square are the same length:

The rhombus



A *rhombus* is like a square except that the interior angles can be any value at all, not just 90°. The square is a special type of rhombus.

Polygons

There is a general name for any *plane* shape which has *straight* sides and is a *closed* shape – a 'polygon'.

Polygon Names	
Number of Sides	**Name**
3	Triangle
4	Quadrilateral
5	Pentagon
6	Hexagon
7	Heptagon
8	Octagon
9	Nonagon
10	Decagon
12	Dodecagon

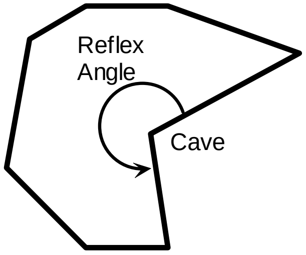
Regular polygons

- All their sides are the same length
- All their interior angles are the same

Irregular polygons

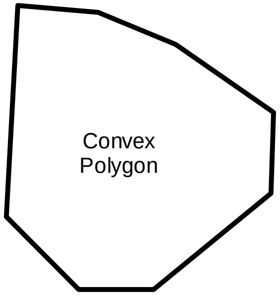
- Has at least two sides of different length
- Has at least two different sized interior angles

Concave polygons



Concave polygons have at least one interior angle that is *larger than 180°*.

Convex polygons



None of the interior angles in a convex polygon are larger than 180°.

Sum of the angles inside a polygon

$$\text{Sum of interior angles} = (n-2)\times 180°$$

where n is the number of sides.

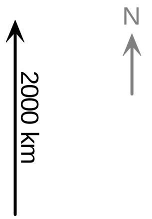
Vectors

Scalars

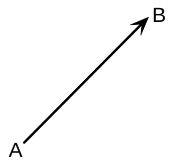
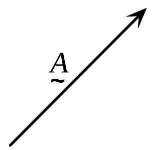
Scalars are things which just have a value – for instance '5' is a scalar quantity – all it has is a value of '5', it doesn't have any other *characteristics* or *properties*.

Vectors

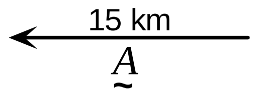
Vectors have both a magnitude value *and* a direction. An everyday example of a vector quantity is if you are describing the movement of a ship or plane. You might say something like, "the plane travelled 2000 km north." You could represent this movement on a diagram by an arrow, like this:



This is a vector quantity because it has a magnitude (2000 km) as well as a direction (north). There are two main ways to talk about a vector:

	
In this case, we can describe this vector as $\overrightarrow{AB}$ or $\underset{\sim}{AB}$. The arrow above the AB or the tilde sign below the AB tells us that it's a vector.	If we just name the whole vector using one letter, then we can describe it using the letter with a tilde sign underneath it to tell us it's a vector, like this: $\underset{\sim}{A}$.

Vector magnitudes



To talk just about the magnitude of this vector, but not the direction, what you can do is write the vector letter down and put a pair of vertical lines around it:

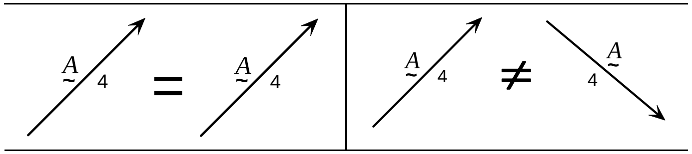
$$\left|\underset{\sim}{A}\right| = 15\,km$$

When we talk about the *entire* vector, then it would look something like this:

$$\underset{\sim}{A} = 15\,\text{km west}$$

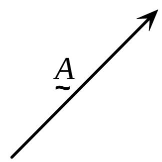
Equal vectors

For *vectors* to be equal, they have to have the same magnitude *and* the same direction:

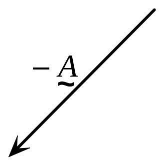


Taking the negative of a vector

If you put a negative sign in front of a vector, you reverse its direction. It keeps the same magnitude however. So the negative of:

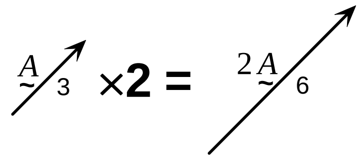


is



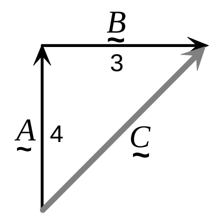
Multiplying a vector by a scalar

If you want to change the magnitude but not the direction of a vector, you can multiply it by a scalar. Any scalar larger than 1 will make the vector longer, any scalar smaller than 1 will make it shorter. For instance:



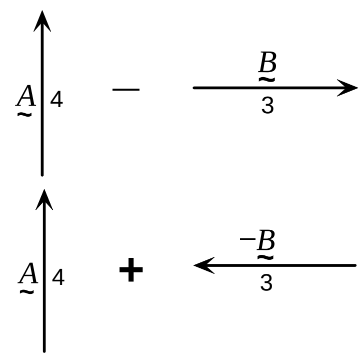
Adding vectors

To add two vectors together, start by drawing the first one. Then, beginning at the arrowhead of the first vector, start drawing your second vector. The answer is the vector from the *start* of your first vector to the *end* of your second vector.



Subtracting Vectors

Do an addition but reverse the vector being subtracted first:



Units

Imperial to Metric

Length

1 inch = 25.4 mm
1 foot = 304.8 mm
1 yard = 914.4 mm
1 mile = 1.609 km
1 international nautical mile = 1.853 km
1 league (land) = 4.828 km
1 league (nautical) = 5.556 km
1 astronomical unit = 149 597 871 km
1 light year = 9 454 254 955 488 km
1 parsec = 30 856 774 878 505 km

Area

1 square inch = 6.452 cm^2
1 square foot = 929.0 cm^2
1 square yard = 0.8361 m^2
1 acre = 4047 m^2
1 hectare = 10000 m^2

Volume

1 cubic inch = 16.39 cm^3
1 cubic foot = 0.02832 m^3
1 cubic yard = 0.7646 m^3

Fluid Volume

1 fluid ounce (US) = 29.57 mL
1 pint (US) = 0.4731 L
1 pint (UK) = 0.5683 L
1 gallon (UK) = 4.546 L
1 gallon (US) = 3.785 L
1 barrel (petroleum) = 159.0 L

Weight

1 ounce = 28.35 g
1 pound = 0.4536 kg
1 metric tonne = 1000 kg
1 US short ton = 907.2 kg
1 stone = 6.350 kg

Temperature

$$\text{Degrees Celsius} = \frac{5}{9}(\text{degrees Fahrenheit} - 32)$$

0 °F = –17.78 °C
32 °F = 0 °C
50 °F = 10 °C
70 °F = 21.11 °C
90 °F = 32.22 °C
110 °F = 43.33 °C

Degrees Celsius = degrees Kelvin – 273.15

273.15 K = 0 °C

Speed

1 mile/hour = 1.609 km/hr
Mach 1 = 1225 km/hr
1 knot = 1.852 km/hr
Speed of light (in air) = 299 792 km/s
Speed of sound (in air) = 1225 km/hr (Mach 1)

Energy

1 calorie (food) = 4.187 kJ
1 ton explosive = 4 184 000 kJ

Pressure

1 bar = 0.9869 atmospheres
1 pascal = 0.000 009 869 atmospheres
1 psi = 0.068 045 957 atmospheres
1 psi = 6.895 kilopascals

Metric Length to Length

10 mm = 1 cm
1000 mm = 1 m
100 cm = 1 m
1000 m = 1 km

Metric Area to Area

100 mm^2 = 1 cm^2
10 000 cm^2 = 1 m^2
1 000 000 m^2 = 1 km^2

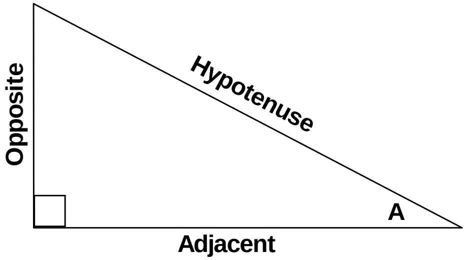
Metric Volume to Volume

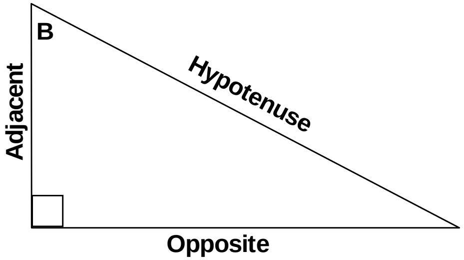
1000mm^3 = 1 cm^3
1 000 000 cm^3 = 1 m^3
1 000 000 000 m^3 = 1 km^3

Cooking

1 metric cup = 250 mL
1 dash = 0.6161 mL
1 drop = 0.05134 mL
1 fifth = 757.1 mL
1 US liquid ounce = 29.57 mL
1 pinch = 0.3081 mL
1 quart (US) = 946.4 mL
1 shot = 29.57 mL
1 metric tablespoon = 15 mL
1 metric teaspoon = 5 mL

Trigonometry





Naming the Sides

There are *three* different names for the three different sides of a right-angled triangle.

The **hypotenuse** is always the longest side.

The other two names, opposite and adjacent, depend on which *angle* you're currently looking at in the triangle.

The **opposite** side is the side opposite the angle we're looking at.

The **adjacent** side is the side of the triangle that *touches* the angle we're looking at, but which is *not* the hypotenuse.

Side Ratios

Trigonometry is all about the *ratio* of the side lengths in the triangle. For the triangle with angle A:

$$\text{cosine A} = \cos \text{A} = \frac{\text{Length of adjacent side}}{\text{Length of hypotenuse side}}$$

$$\text{sine A} = \sin \text{A} = \frac{\text{Length of opposite side}}{\text{Length of hypotenuse side}}$$

$$\text{tangent A} = \tan \text{A} = \frac{\text{Length of opposite side}}{\text{Length of adjacent side}}$$

There's an easy way to remember what all these ratios are – **SOH CAH TOA**

SOH = **(S)**in : **(O)**pposite over **(H)**ypotenuse

CAH = **(C)**os : **(A)**djacent over **(H)**ypotenuse

TOA = **(T)**an : **(O)**pposite over **(A)**djacent

Check with Pythagoras' Theorem

You can check side length answers with Pythagoras' Theorem:

$$\text{Hypotenuse}^2 = \text{opposite}^2 + \text{adjacent}^2 \text{ ???}$$

Say the hypotenuse is 8, and you work out the adjacent side is 6.128 and the opposite side is 5.142.

$$8^2 = 5.142^2 + 6.128^2 \text{ ???}$$

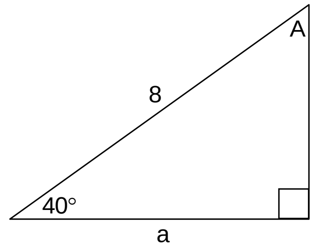
$$64 = 26.44 + 37.55 \text{ ???}$$

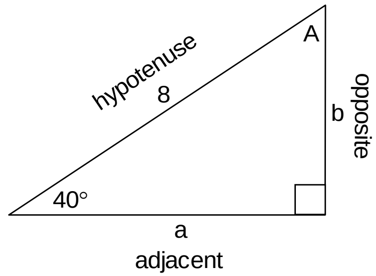
$$64 = 63.99 \text{ ???}$$

$$64 \approx 63.99$$

The sides match up so you've probably got the right answer.

Using Trigonometry to work out side lengths





Say we want to find how long side 'a' is. What we can do is write a trigonometric ratio which has 'a' in it. We only know one side length – we know the hypotenuse is 8 long. What we want is a trigonometric ratio that involves both the side we're trying to work out (side a) and the side we already know.

So since we're using the 40° angle, we want a ratio which involves the adjacent side and the hypotenuse. Let's go through our SOH CAH TOA:

(S)in is the (O)pposite over the (H)ypotenuse, so it won't work.

(C)os is the (A)djacent over the (H)ypotenuse – bingo, that's what we want.

So we can write a trigonometric ratio using *cosine*:

$$\cos 40° = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{a}{8}$$

$$\cos 40° = \frac{a}{8}$$

$$a = 8\cos 40°$$

$$a \approx 6.1$$

Statistics

Recording Data – Tally Table

Example: Election votes

Candidate	Tally of votes	Total
Bob	𝍸 𝍸 𝍸 𝍸 III	23
Jane	𝍸 𝍸 𝍸 II	17
Sally	𝍸 𝍸 III	13
Tom	𝍸	5

The pie chart / sector graph

To draw the pie chart, you need to know what *fraction* of the total votes each candidate got. To work this out, you need to work out how many votes were collected in total *for all the candidates*:

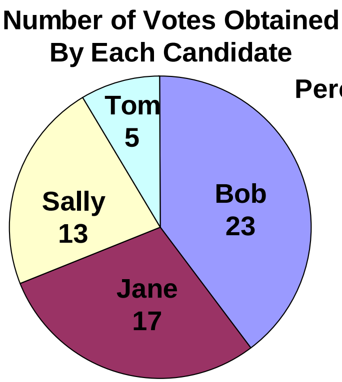
$$\text{Total votes} = 23 + 17 + 13 + 5$$

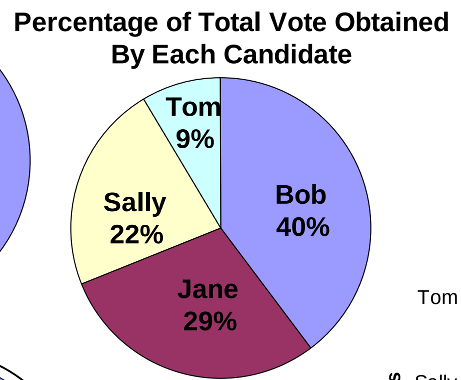
$$\text{Total votes} = 58$$

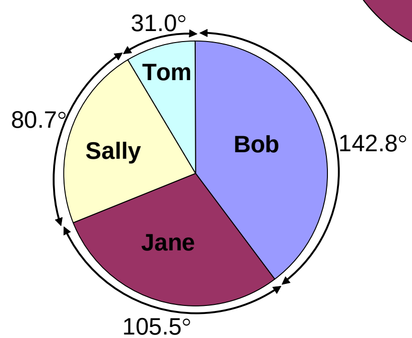
Candidate	Fraction of Total Votes
Bob	$\frac{23}{58} = 0.40 = 40\%$
Jane	$\frac{17}{58} = 0.29 = 29\%$
Sally	$\frac{13}{58} = 0.22 = 22\%$
Tom	$\frac{5}{58} = 0.086 = 8.6\%$

If yu rotate around a circle completely, you will have rotated through 360°. We need to draw wedges representing the fraction of votes each candidate got.

Candidate	Wedge Angle
Bob	$\frac{23}{58} \times 360° = 142.8°$
Jane	$\frac{17}{58} \times 360° = 105.5°$
Sally	$\frac{13}{58} \times 360° = 80.7°$
Tom	$\frac{5}{58} \times 360° = 31.0°$

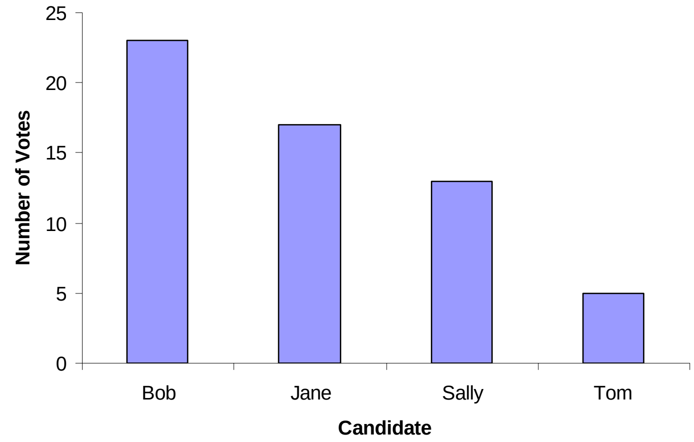






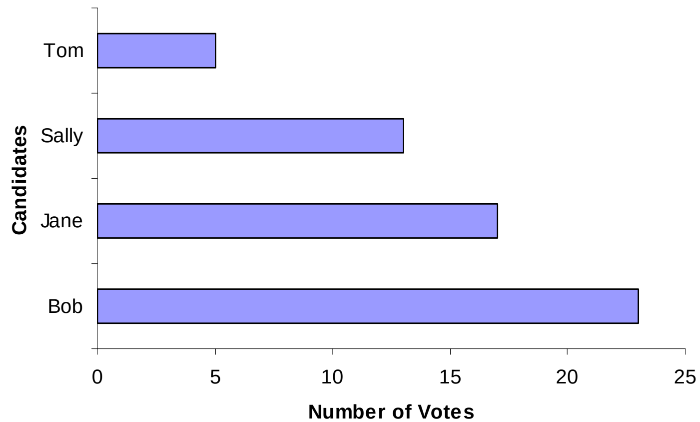
Column Graph

The graph has two *axes*. The horizontal axis shows the different *categories* in the statistical problem – in our case, the different candidates people could vote for. The vertical axis shows the *quantities* of the things we're counting or surveying – in this case, the *votes* for each of the candidates.



Bar Graph

A bar graph's basically the same as a column graph, except that it has its two axes swapped over, which means that the columns become horizontal bars. Here's what a bar graph would look like for the vote data:

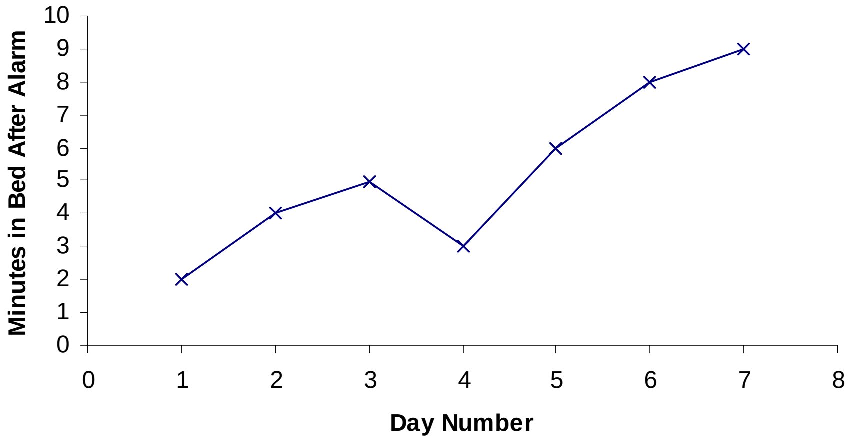


More Statistics

Line Graphs

- Used a lot to show how something is changing over time, and can even be used to predict the future (carefully).
- Can also show *trends*. A trend is the general direction in which something is heading or tending towards. For instance, the population of the world has a trend – it is *increasing*.
- Usually have two axes - horizontal axis which shows categories or time, vertical axis which usually shows a quantity.
- *Adjacent* (next to each other) categories on the horizontal axis have to be somehow linked to each other.
- This table shows some data about how long a student Jason has stayed in bed after his alarm has gone off over the course of a week.

Day	1	2	3	4	5	6	7
Minutes in bed after alarm	2	4	5	3	6	8	9

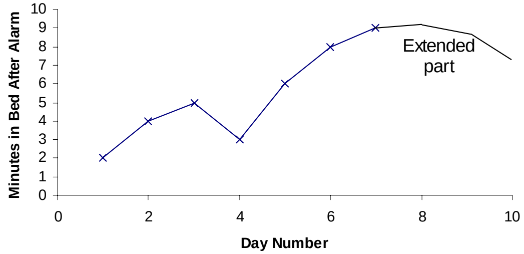


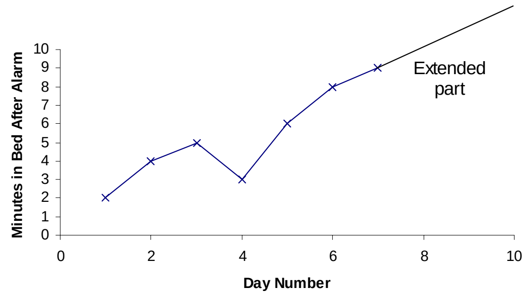
Trends

- Trends are overall characteristics of which 'way' the data is heading – for instance whether the quantities are *in general* increasing or decreasing.

Extrapolating

- Extrapolation is used to try and predict data outside of the data you're given.
- Extrapolating a line graph is what happens when you *extend* the line graph in either direction, to the left or to the right.
- Which way you pick to extend the line when you extrapolate isn't as important as *justifying* why you extended it in that way.



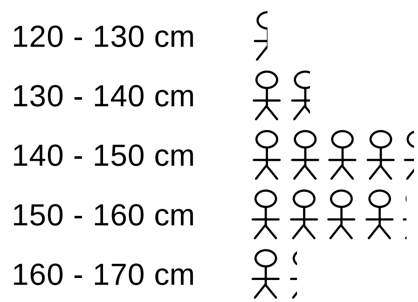


1st graph: Based on the trend from days 4 to 7, which indicates that Jason's stay in bed time is increasing, but increasing by a smaller amount each day, the line was extended in this way.

2nd graph: Based on the overall trend in the graph, which is an increasing trend, the line was extended in an increasing manner corresponding with the average increase of the existing line.

Pictographs

- Pictographs use pictures to represent information.
- Each stick figure represents ten school students
- Half a stick figure represents five students
- Pictographs can reduce accuracy, but are often more visually attractive
- Reinforce what the data is about – in this case the number of students (hence stick figures)



For the following section, the ages of 13 students in a class are used:

15, 14, 14, 15, 15, 15, 15, 16, 15, 14, 15, 15, 15

Mean

The *mean* is the *average* of all the data. There is a special symbol for the mean of a set of numbers – it's an 'x' with a horizontal line above it: $\overline{X}$.

$$\bar{x} = \frac{\sum x}{n} \text{ where } \sum x = \text{sum of all numbers}$$

For the class:

$$\bar{x} = \frac{\sum x}{n}$$

$$\bar{x} = \frac{15+14+14+15+15+15+15+16+15+14+15+15+15}{13}$$

$$\bar{x} = 14.85$$

Mean

The median value is the *middle* value in the list once you've ordered them:

14, 14, 14, 15, 15, 15, **15**, 15, 15, 15, 15, 15, 16

15 is the median value. If you have an even number of values, you take the average of the middle two numbers. So if there were only 12 ages:

14, 14, 14, 15, 15, **15**, **15**, 15, 15, 15, 15, 16

The median is the average of the two middle values 15 and 15. So the median is 15.

Mode

The mode is the most commonly occurring number in the entire list. Can be more than one mode.

Even More Statistics

Create a frequency table for the original 13 ages:

Student age	Number of students with that age
14	3
15	9
16	1

The mode is 15.

Mean Versus Median

The mean generally seems to take into account *all* the values in a data set, which means that one *outlier* (an outlier is a value that is significantly different to most of the other values) can significantly change the mean.

The median is much better at effectively *ignoring* outliers.

Because of the median's ability to ignore outlying values, it is often regarded as a more *robust* measure, because it is focused around the middle values and ignores extreme values on either side.

Median and mean are used a lot in real estate.

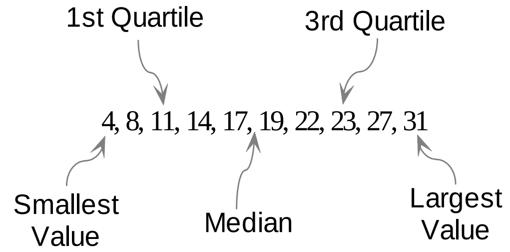
Box and Whisker Plots

Start with data: 11, 23, 14, 27, 8, 4, 31, 22, 17, 19

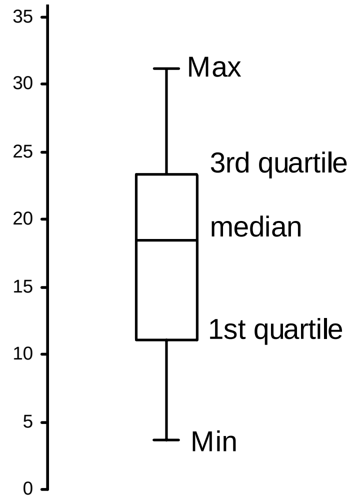
Order it: 4, 8, 11, 14, 17, 19, 22, 23, 27, 31

The m**edian** is the middle value - the average of 17 and 19 – 18.

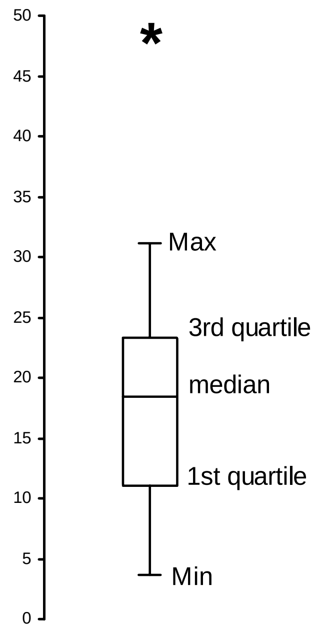
The **1st quartile** value is the median value *out of the lower half of the data*, the **3rd quartile** value is the median value *out of the upper half of the data*:



- Short horizontal lines for minimum and maximum.
- Long horizontal lines for 1st and 3rd quartiles and median, with two vertical lines making the 'box'.

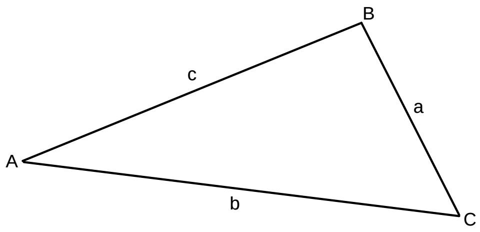


Outliers can be represented by an '*'. Here a data value of '49' is represented as an outlier.



Cosine and Sine Rule

The Cosine Rule

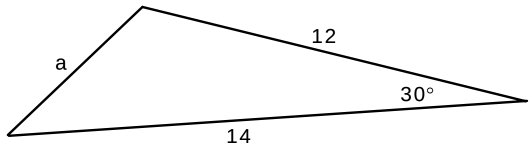


The cosine rule says that:

$$a^2 = b^2 + c^2 - 2 \times b \times c \times \cos A$$

In words, this means that, "*a* squared is equal to the sum of the squares of the other two sides in the triangle, minus two times the *product* of the other two sides times cos of the angle opposite *a*."

The cosine rule **does not need a right angle** in the triangle to work.



$$a^2 = b^2 + c^2 - 2 \times b \times c \times \cos A$$

$$a^2 = 12^2 + 14^2 - 2 \times 12 \times 14 \times \cos 30°$$

$$a = \pm\sqrt{49.02}$$

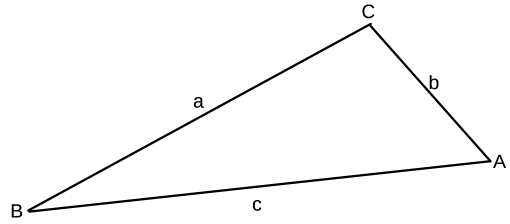
$$a = \text{(about) } 7 \textit{ units}$$

Don't forget to take the square root! Note that I have only taken the positive answer, as a negative side length does not make sense in this situation.

In general the cos rule is used to find the length of one side, when you know:

- the length of the two other sides in the triangle, and
- the angle between these two other sides.

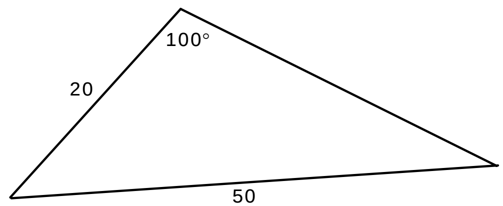
The Sine Rule



The sine rule states that:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

The triangle **does not** have to have a right angle in it for this rule to work.



Let a = the side of length 50

Let A = the angle opposite 'a' (100°)

Let b = the side of length 20

Let B = the angle opposite 'b'

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\frac{50}{\sin 100°} = \frac{20}{\sin B}$$

Invert both sides

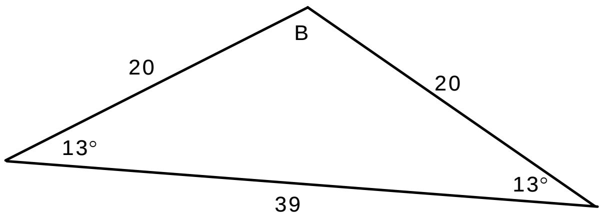
$$\frac{\sin 100°}{50} = \frac{\sin B}{20}$$

$$20 \times \frac{(\sin 100°)}{50} = \sin B$$

$$\sin B = 0.3939$$

$$B \approx 23.2°$$

Problems with the Sine Rule



If you used the sine rule to solve for the unknown angle in this triangle, you'd get:

$$\frac{20}{\sin 13°} = \frac{39}{\sin B}$$

$$\frac{\sin 13°}{20} = \frac{\sin B}{39}$$

$$\frac{39 \times (\sin 13°)}{20} = \sin B$$

$$B = \sin^{-1}\left[\frac{39 \times \sin 13°}{20}\right]$$

$$B = 26°$$

26 degrees seems way too small an angle – and it is. There is a flaw in the sine rule based around the fact that:

$$\sin(180° - \theta) = \sin \theta$$

Whenever you get an angle using the sine rule, you must check to see whether it makes physical sense – if it doesn't then you must use the above rule to find the other possible answer:

$$\sin(180° - 26°) = \sin 26°$$

$$\text{So } \sin 154° = \sin 26°$$

An angle of 154 degrees is much more reasonable for angle B. It also means the sum of angles inside the triangle is 180°, so it looks like the correct answer.

Simultaneous Equations

Unknowns

To work out the unique values of the unknowns in equations, you need at least as many *different* pieces of information as there are unknown values.

The substitution method

$$x+2y=5$$
$$2x-3y=-4$$

To use this method, just pick one of the equations: $x+2y=5$.

Pick one of the two unknowns and substitute something else for it.

Say we pick 'x'. We want to replace the 'x' with what it is equal to.

We can rearrange our second equation, so that it is in the form of "x = …":

$$2x-3y=-4$$
$$2x-3y+3y=-4+3y$$
$$2x=-4+3y$$
$$x=\frac{-4+3y}{2}$$
$$x=-2+\frac{3}{2}y$$

So now we have an expression that says what x is equal to. We can substitute this into our first equation:

Replace this x (in other words, substitute for x)...

$$x+2y=5$$

$$x=-2+\frac{3}{2}y$$

...with what we know x is equal to

To get this: $-2+\frac{3}{2}y+2y=5$

Now we've only got *one* unknown in this equation, so we can solve for the value of y:

$$-2+\frac{3}{2}y+2y=5$$
$$-2+3\frac{1}{2}y=5$$
$$3\frac{1}{2}y=7$$
$$\frac{7}{2}y=7$$
$$7y=14$$
$$y=2$$

Use our x = equation from earlier:

$$x=-2+\frac{3}{2}y$$
$$x=-2+\frac{3}{2}\times 2$$
$$x=-2+3$$
$$x=1$$

The Elimination Method

The elimination method is all about *getting rid* of one of the unknowns in the equations, by *adding* or *subtracting* the equations.

$$3a+2b=3$$
$$2a+b=1$$

To get rid of one of the unknowns, you need to change one of the equations so that it has exactly the same unknown term in it as the other. Can multiply the 2nd equation by 2:

	$2a+b=1$
×	2
	$4a+2b=2$

Now subtract the 2nd equation from the first:

	$3a+2b=3$
–	$4a+2b=2$
	$-a=1$

If –a is equal to 1, this must mean a = –1. Now easy to solve for the value of 'b' using the first equation:

$$3a+2b=3$$
$$3\times -1+2b=3$$
$$-3+2b=3$$
$$2b=6$$
$$b=3$$

Quadratic Equations

Quadratic equations are second degree equations which always have a variable raised to the power '2'.

$x^2 = 4$ is a quadratic equation, because 'x' is a variable and it's raised to the power '2'. It could also be written $x^2 - 4 = 0$.

$x^2 + 2x - 5.5 = 0$ is also a quadratic equation, because 'x' is a variable, and it's raised to the power '2'. This equation however also has a term where 'x' is raised to the power '1'. It's still a quadratic equation though.

$x^3 + 2x^2 - 5 = 0$ is not a quadratic equation, because it has a variable raised to a power higher than '2'.

General Form

The *general* form of a quadratic equation is:

$$ax^2 + bx + c = 0$$

The letters a, b, and c are just simply numbers. They can be fractions or decimals, negative or positive numbers. Any equation that fits this general form **or** can be rearranged to fit this general form is a quadratic equation.

For instance, the following is a quadratic equation:

$$x^2 = 4$$

It doesn't look the same simply because this equation has 'b' being 0. It is really:

$$(1 \times x^2) + (0 \times x) - 4 = 0$$

Quadratic Equations

Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Use it to solve: $x^2 + 3x + 2 = 0$

For this equation, a = 1, b = 3 and c = 2. So:

$$x = \frac{-3 \pm \sqrt{3^2 - 4 \times 1 \times 2}}{2 \times 1}$$

$$x = \frac{-3 \pm \sqrt{1}}{2}$$

$$x = \frac{-3 + \sqrt{1}}{2}, \frac{-3 - \sqrt{1}}{2}$$

$$x = -1, -2$$

Note how a '±' sign means that there are actually two solutions, one for when it is a '+', and one for when it is a '–'.

Solving By Factorising

$$2x^2 + 6x + 4 = 0$$

a = 2, b = 6 and c = 4. Think of ways to get $2x^2$:

(2x + …) (x + …)

Think of two numbers that multiply together to give c (which is 4):

2 and 2, –2 and –2, 1 and 4, –1 and –4

Let's try 2 and 2:

$$(2x + 2)(x + 2)$$

To check whether you have a correct guess, multiply the brackets out:

$$(2x + 2)(x + 2)$$
$$= 2x^2 + 4x + 2x + 4$$
$$= 2x^2 + 6x + 4$$

If not, try different combinations of numbers.

Picking The Correct Solution

Now say you've just solved a quadratic equation and you've got two answers, t = 5, –1. For this particular problem pretend that the solutions represent how long in seconds it takes a rocket to reach a certain height. The solution t = 5 is fine, but what does the solution t = –1 mean? This suggests that the rocket reaches a certain height before it is launched!

Obviously this cannot be true, so only the solution t = 5 should be used. This can happen quite often – you must pick the 'sensible' solutions. (t = –1 is a mathematical solution that doesn't make any sense when you use it in a real life application).

Only one answer to the square root of a number by itself

There is only one answer when you take the square root of a number just by itself:

$$\sqrt{4} = 2 \text{ only}$$

You only get the two 'plus' and 'minus' answers when you've got the square of a variable involved, and you need to take the square root of both sides, like this:

$$x^2 = 4$$
$$x = \pm\sqrt{4}$$
$$x = \pm 2$$

Proportional or direct relationships

A proportional or direct relationship is one where the value of one variable is directly related to the value of another. For instance, a very simple direct relationship is:

$$y = 2x$$

Constant or Proportionality

In order for a relationship to be proportional, you need to be able to write it in this general form:

$$y = kx$$

'k' is called the *constant of proportionality*, and can be any value you need apart from zero.

You can also have direct relationships where one variable varies as the *square* of the other variable, like this one for instance:

$$y = kx^2$$

In this case we'd say that y varies directly as x^2.

The Proportional Sign

When one variable is directly related to another, you can use the *proportionality symbol* to describe the relationship. The symbol is a '$\propto$'. So if we write something like…

$$a \propto b$$

…what we mean is that the value of 'a' is proportional to the value of 'b', and that we can write a *quantitative* (one involving actual numbers and values) relationship using just a *constant of proportionality*, like this:

$$a = kb$$

Same goes for 'proportional to the square' relationships, for instance for $y = kx^2$:

$$y \propto x^2$$

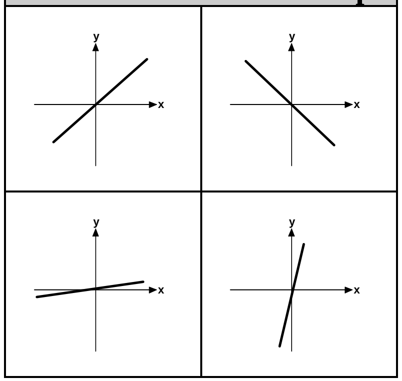
Direct Linear Variation

When one variable is proportional to some constant times the other variable, this is known as direct *linear* variation.

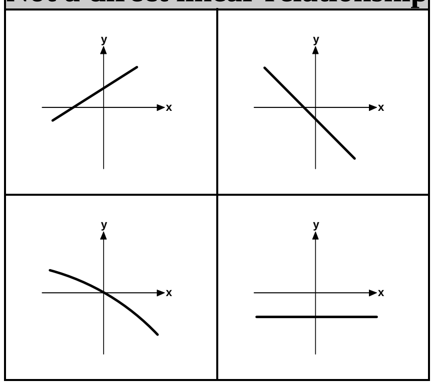
Graphs of a direct linear relationship have these characteristics:

- The line is a straight one
- This straight line always passes through the *origin* of the graph (the 0, 0 point where the axes usually intersect)

Direct linear relationship



Not a direct linear relationship



From an Equation

Get the equation into a form where there's one variable on each side of the equals sign. Then, if both variables are raised to the power '1', and you've got no other terms, then you've got yourself a linear relationship:

Direct linear relationship	Not a direct linear relationship
$a = 5b$ $t = 7.5s$ $u = \frac{c}{23}$	$x = \frac{7}{y}$ $b = a^2$ $e = \sqrt{f}$

From a Table

It's pretty easy to tell if there is a direct linear relationship in a table of data. Just assume that there is one, and work out the value of the proportionality constant from that. Then use the formula to check the other values in the data set.

x	2	3	4	5
y	5	7.5	10	12

$$y = kx$$
$$5 = k \times 2$$
$$k = 5 \div 2$$
$$k = 2.5$$

so $y = 2.5x$

Then use this equation to check the other values in the table. For x = 3, 4, 5:

$$y = 2.5 \times 3 \qquad y = 2.5 \times 4 \qquad y = 2.5 \times 5$$
$$y = 7.5 \qquad y = 10 \qquad y = 12.5$$

12.5 does *not agree* with the y value of '12' in the table - the data in the table is not part of a direct linear relationship.

Probability

$P(E)$

This simple statement just says something like "The probability (P) of event E occurring."

The probability of an event happening is calculated in this way:

$$\text{P}(\text{E}) = \text{Event probability} = \frac{\text{Number of outcomes of that event}}{\text{Total number of outcomes}}$$

Probability of picking a pink marble:

$$\text{P}(\text{E}) = \frac{2}{10} = \frac{1}{5} = 0.2 = 20\%$$

Handy hint – don't double count

If you pick a card at random from a normal 52 card pack, what is the chance that you will pick a red card *or* ace from the pack?

$$\text{P}(\text{E}) = \frac{28}{52} = \frac{7}{13} = 0.5385 \approx 54\%$$

There are 26 outcomes that result in a red card being picked, *and a further two outcomes* that result in an ace – two of the aces have *already been counted* in the red cards. Don't double count the red aces.

Not Probabilities

What is a 'not' probability? It's the chance of a certain event *not* happening. The probability of the event happening plus the probability of the event not happening adds together to give 1:

$$\text{P}(\text{E}) + \text{P}(\text{not E}) = 1$$

For example, tomorrow's weather:

$$\text{P}(\text{rain}) + \text{P}(\text{no rain}) = 1$$

Probability Terminology

Example: Bob is picking a single marble out of a bag without looking. There are 10 marbles in the bag – 4 red ones, 3 blue ones, 2 pink ones and 1 white one.

Trial / experiment

This is all about the thing that is happening – in this case the trial or experiment is Bob picking a marble out of the bag.

Outcome

This is the result of what happens when we do the trial or experiment. In this case, the outcome of Bob picking a marble is that he will get a marble with a certain colour.

Outcome space

The *outcome space* is an imaginary space that contains all the possible *outcomes* that you could get from an experiment. An outcome in this experiment is Bob picking a marble of a certain colour. Well, what colours could he get? He could get *red*, *blue*, *pink* and *white*.

Event

Event is used to describe something that happens which you might be interested in. For instance, you might be interested in when Bob picks out a blue marble. The event you're interested in is, "Bob picking out a blue marble."

Probability of an event occurring

We can use the letter 'P' to indicate "probability of", and then inside some brackets after the 'P' write what event we're calculating the probability for. The event is usually represented by the letter 'E'.

Certainties and impossibilities

A certainty is something that has a 100 % chance of happening – it will happen *for sure*:

$$\text{P}(\text{certainty}) = 1 \text{ or } 100\%$$

Impossibilities are the opposite of certainties.

$$\text{P}(\text{impossibility}) = 0 \text{ or } 0\%$$

Set Language

$$\text{Pr}(E) = \frac{n(E)}{n(A)}$$

- $\text{Number of outcomes of that event} = n(E)$
- $\text{Total number of outcomes} = n(A)$

Subsets

$E \subset A$

The $\subset$ symbol means "is a subset of". This means that the set E is a *subset* (smaller part of) the set A.

'Not' sets

Using proper mathematical notation, we can write "Not E" as:

$$E'$$

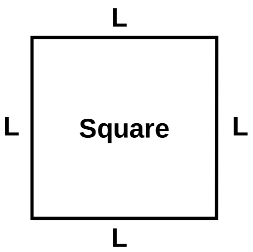
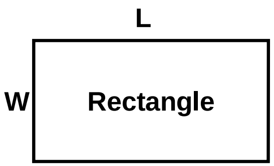
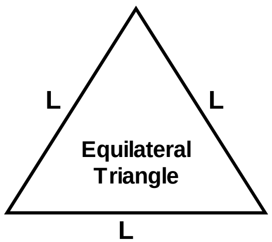
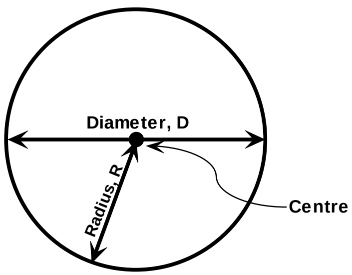
Complementary sets

The probability of event E occurring and the probability of it *not* occurring adds up to 1, or 100%. We can talk about these as two sets. Set E contains all the outcomes that result in E happening. The other set, E', contains all the outcomes that *don't* result in E happening.

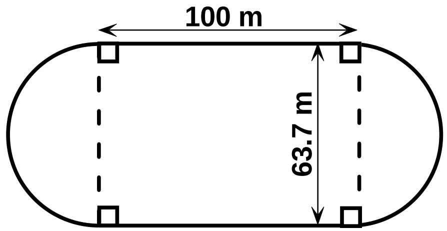
These are 'complementary' sets. They *complement* each other to make up a total probability of 1.

Perimeters of 2D Shapes

Perimeters

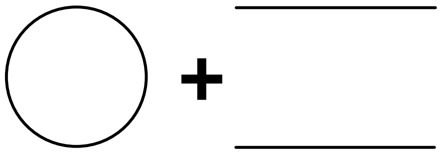
	Perimeter = 4L
	Perimeter = 2(L + W)
	Perimeter = 3L
Circle 	Perimeter = πD Or Perimeter = $2\pi R$

Running Track Perimeter Question



You can work out the whole perimeter by doing a sort of sum of shape perimeters like this:

Which, since the two half circles make a whole circle, is the same as:



$$\text{Perimeter of circle} = \pi D$$

$$\text{Perimeter of circle} = 3.14 \times 63.7m$$

$$\text{Perimeter of circle} = 200.0m$$

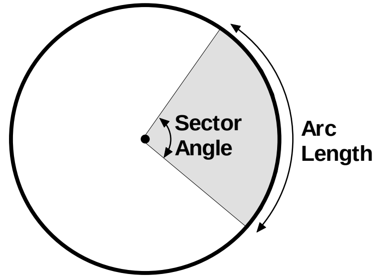
$$\text{Total perimeter} = \text{circle perimeter} + \text{straight bits length}$$

$$\text{Total perimeter} = 200.0m + 200.0m$$

$$\text{Total perimeter} = 400.0m$$

Length of an arc

A sector is a chunk of a circle like a wedge of cake. The sector in the diagram has a *sector angle* of about 95°. The curvy bit of the sector is called the *arc length*.



The fraction that the sector angle is of 360°, is also the fraction that the arc length is of the full circumference. In mathematical terms this is:

$$\text{Arc length} = \frac{\text{Sector angle}}{360°} \times \pi D$$

Using π

Pi is a special number because you can keep writing down digits to the right of the decimal place forever:

π =3.14159265358979323846264338327950288 41971693993751058209 … and so on and so on…

Now when you're doing an exam, you've got two choices. You can either use a memorised version of pi to do your calculations – most maths teachers are happy if you use "pi to two decimal places" – 3.14 in other words. The other option is to use the pi button on your calculator – when you press the pi button, it's as if you'd entered pi to about 8 or 10 decimal places yourself.

Matrices

What is a Matrix

A simple matrix is a two-dimensional clump of numbers. It has a length and a height. For instance, this is a *two by three* matrix:

$$\begin{bmatrix} 4 & 17 & -5 \\ 22 & -12 & 7 \end{bmatrix}$$

The first dimension mentioned is the number of rows – two. The second dimension mentioned is the number of columns – three.

The order of a matrix

You always describe the number of *rows* first, then the number of *columns*. This matrix has two rows and three columns, which is why it's described as a two by three matrix. This is often called the *order* of the matrix – the number of rows by the number of columns.

Two rows

$$\begin{bmatrix} 4 & 17 & -5 \\ 22 & -12 & 7 \end{bmatrix}$$

Three columns

$$\begin{bmatrix} 4 & 17 & -5 \\ 22 & -12 & 7 \end{bmatrix}$$

In a two by three matrix, there are six *elements* – cells or places where a value is.

Describing an element

The 22 could be described in this way:

$$A_{21} = 22$$

This says that the value in row 2, column 1 of matrix A is equal to 22. To talk about the '–5' element we'd say:

$$A_{13} = -5$$

Adding and subtracting matrices

You can only add or subtract one matrix with/from another if they have the same order – the same number of rows *and* the same number of columns. To actually do the calculation, all you do is perform the addition or subtraction on each pair of *corresponding* elements (the elements in the same position as each other):

$$\begin{bmatrix} 2 & 7 \\ -15 & 4 \end{bmatrix} + \begin{bmatrix} -5 & 14 \\ -7 & -8 \end{bmatrix}$$

$$= \begin{bmatrix} 2+-5 & 7+14 \\ -15+-7 & 4+-8 \end{bmatrix}$$

$$= \begin{bmatrix} -3 & 21 \\ -22 & -4 \end{bmatrix}$$

Multiplying matrices

To do multiplication, you start at the *top row* of the first matrix, and multiply each of these elements by the ones down the *first column* of the second matrix.

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix} \times \begin{bmatrix} 7 & 8 \\ 9 & 10 \end{bmatrix} = \begin{bmatrix} 1\times7+2\times9 & ? \\ ? & ? \\ ? & ? \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix} \times \begin{bmatrix} 7 & 8 \\ 9 & 10 \end{bmatrix} = \begin{bmatrix} 1\times7+2\times9 & 1\times8+2\times10 \\ 3\times7+4\times9 & ? \\ ? & ? \end{bmatrix}$$

The number of *rows* in your new matrix will be equal to the number of rows in your *first* matrix. The number of *columns* in your new matrix will be equal to the number of columns in your *second* matrix. So its order will be 3×2.

The next step in the calculation is to stick with the same row in the first matrix, but multiply it by the elements in the *second* column of the second matrix, like this:

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix} \times \begin{bmatrix} 7 & 8 \\ 9 & 10 \end{bmatrix} = \begin{bmatrix} 1\times7+2\times9 & 1\times8+2\times10 \\ ? & ? \\ ? & ? \end{bmatrix}$$

Now we've finished with the first row in both our first matrix *and* our answer matrix. We move on to the second row of our first matrix. The first step is to multiply it by the first *column* of the second matrix:

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix} \times \begin{bmatrix} 7 & 8 \\ 9 & 10 \end{bmatrix} = \begin{bmatrix} 1\times7+2\times9 & 1\times8+2\times10 \\ 3\times7+4\times9 & ? \\ ? & ? \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix} \times \begin{bmatrix} 7 & 8 \\ 9 & 10 \end{bmatrix} = \begin{bmatrix} 1\times7+2\times9 & 1\times8+2\times10 \\ 3\times7+4\times9 & 3\times8+4\times10 \\ 5\times7+6\times9 & 5\times8+6\times10 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix} \times \begin{bmatrix} 7 & 8 \\ 9 & 10 \end{bmatrix} = \begin{bmatrix} 25 & 28 \\ 57 & 64 \\ 89 & 100 \end{bmatrix}$$

Multiplying and dividing by a scalar

A scalar is a single number, rather than a group of numbers like in a matrix. All you do is perform the multiplication / division individually on each element in the matrix.

$$\begin{bmatrix} 14 & 19 \\ 7 & 6 \end{bmatrix} \times 4 == \begin{bmatrix} 14\times4 & 19\times4 \\ 7\times4 & 6\times4 \end{bmatrix} = \begin{bmatrix} 56 & 76 \\ 28 & 24 \end{bmatrix}$$

Identity Matrix

The term ‘identity matrix’ is used to describe any *square* matrix (same number of rows as columns) which has ones down its primary diagonal, and zeros everywhere else. The primary diagonal starts at the top left corner of the matrix and continues down to the bottom right corner. For instance, these are both identity matrices:

$$I_2=\begin{bmatrix}1&0\\0&1\end{bmatrix},\ I_3=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$$

The *subscript* of an I (a little number to the right and below the ‘I’) is used to describe how large the identity matrix is.

Any matrix you multiply by an *identity* matrix, you just get the original matrix again as your answer.

$$\Rightarrow\begin{bmatrix}1&2\\3&4\end{bmatrix}\times\begin{bmatrix}1&0\\0&1\end{bmatrix}$$
$$=\begin{bmatrix}1\times1+2\times0&1\times0+2\times1\\3\times1+4\times0&3\times0+4\times1\end{bmatrix}$$
$$=\begin{bmatrix}1&2\\3&4\end{bmatrix}$$

The Inverse of a Matrix

$$A=\begin{bmatrix}3&5\\2&4\end{bmatrix}$$

The inverse of this matrix is the matrix you need to multiply it by to get the identity matrix.

More Matrices

Finding the matrix inverse

$$A=\begin{bmatrix}a&b\\c&d\end{bmatrix}$$

We can find the inverse like this:

$$A^{-1}=\frac{1}{ad-bc}\begin{bmatrix}d&-b\\-c&a\end{bmatrix}$$

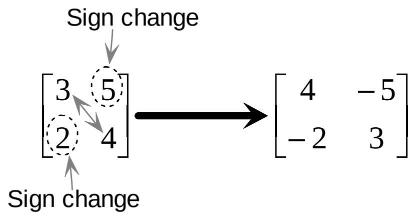
The ‘ad – bc’ part in the denominator of the fraction is called the *determinant* of the matrix A, and is often written as $\det(A)$ or $|A|$:

$$A^{-1}=\frac{1}{\det(A)}\begin{bmatrix}d&-b\\-c&a\end{bmatrix}$$
$$\det(A)=ad-bc$$

Work out determinant for the original A matrix:

$$A=\begin{bmatrix}3&5\\2&4\end{bmatrix}$$
$$\det(A)=ad-bc$$
$$a=3\quad b=5\quad c=2\quad d=4$$
$$\det(A)=3\times4-5\times2$$
$$\det(A)=2$$

Swap around and change signs of the elements in matrix A:



Inverse of A is:

$$A^{-1}=\frac{1}{\det(A)}\begin{bmatrix}4&-5\\-2&3\end{bmatrix}^{-1}=\frac{1}{2}\times\begin{bmatrix}4&-5\\-2&3\end{bmatrix}$$
$$A^{-1}=\begin{bmatrix}2&-2.5\\-1&1.5\end{bmatrix}$$

Check whether when we multiply A by it we really do get the identity matrix:

$$\Rightarrow A\times A^{-1}$$
$$=\begin{bmatrix}3&5\\2&4\end{bmatrix}\begin{bmatrix}2&-2.5\\-1&1.5\end{bmatrix}$$
$$=\begin{bmatrix}3\times2+5\times-1&3\times-2.5+5\times1.5\\2\times2+4\times-1&2\times-2.5+4\times1.5\end{bmatrix}$$
$$=\begin{bmatrix}1&0\\0&1\end{bmatrix}$$

Equations into Matrices

$$\begin{matrix}3x+5y=31\\2x+4y=24\end{matrix}\quad\text{becomes}\quad\begin{bmatrix}3&5\\2&4\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}31\\24\end{bmatrix}$$

Multiplying both sides by the inverse of the first matrix gives you a very useful result.

$$\begin{bmatrix}2&-2.5\\-1&1.5\end{bmatrix}\times\begin{bmatrix}3&5\\2&4\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}2&-2.5\\-1&1.5\end{bmatrix}\times\begin{bmatrix}31\\24\end{bmatrix}$$
$$\begin{bmatrix}2\times3-2.5\times2&2\times5-2.5\times4\\-1\times3+1.5\times2&-1\times5+1.5\times4\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}2\times31-2.5\times24\\-1\times31+1.5\times24\end{bmatrix}$$
$$\begin{bmatrix}1&0\\0&1\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}2\\5\end{bmatrix}$$
$$\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}2\\5\end{bmatrix}\quad\text{so}\quad\begin{matrix}x=2\\y=5\end{matrix}$$

Logic

The basic part of logic in mathematics is quite simple. It is all built around simple *binary* statements or *propositions*. These statements are ones that can either be true *or* false. Here's an example of a proposition:

My name is Michael

This statement can either be true or false. My name can't 'sort of' be Michael, it either is or it is not. Here's an example of a statement which *isn't* a binary statement:

How many names do I have?

This statement is a question. It can't be true or false, it needs a *quantitative* answer (i.e. the number of names I have).

Logic Connectors

There are five major *connectors* that you can use in logic to write *logic sentences*.

OR connector

The $\vee$ symbol is the 'OR' symbol – it means at least *one* of the two things on either side of it will happen. Remember the 'or both' part – it's commonly forgotten.

This afternoon I will play tennis OR I will read (or both)

If I used 't' to describe tennis, and 'r' to describe reading, then I could rewrite this statement:

$$t \vee r$$

AND connector

This afternoon I will play tennis AND I will read

The mathematical symbol for 'AND' is $\wedge$:

$$t \wedge r$$

IMPLIES connector

Jason goes to primary school – this implies he is less than 15 years old

It's a bit trickier to analyse whether an imply statement is true or false. There are three possibilities:

- The first part of the statement is true, and the second part is true as well. In that case the *implication* is *true*.
- The first part of the statement is true, but the second part is false. In that case the implication is *false*.
- The first part of the statement is false. In this case, the implication is *always* true. This is sometimes confusing for people. Whether the second part of the statement is true can only be judged when the first part is true – the second part *leads on* from the first part. So the convention is that when the first part is false, we just say that the whole meaningless implication is *true*.

The mathematical symbol for IMPLIES is an arrow with two lines – $\Rightarrow$. If we use 'PS' for primary school and 'K' for kid under 15 years old, the mathematical way to write the last statement is:

$$PS \Rightarrow K$$

EQUIVALENCE connector

I will play tennis this afternoon if I read a book, and I'll read a book if I play tennis.

The mathematical symbol for EQUIVALENCE is a double headed arrow – $\Leftrightarrow$.

$$t \Leftrightarrow r$$

NOT connector

The logic symbol for this NOT connector is '~'. So if I start with a statement like:

I will play tennis this afternoon

I can convert this into a mathematical statement just like this:

$$t$$

If I put a NOT symbol in front of this, I reverse the statement:

$$\sim t$$

This reverses the statement so it is now:

I will not play tennis this afternoon

Truth Tables

Truth tables are a way of writing down all the possible combinations of statements and saying whether the whole combined statement is true or false.

OR		
A	**B**	$A \vee B$
True	True	True
True	False	True
False	True	True
False	False	False

More Logic

More Truth Tables

AND		
A	**B**	$A \wedge B$
True	True	True
True	False	False
False	True	False
False	False	False

IMPLIES		
A	**B**	$A \Rightarrow B$
True	True	True
True	False	False
False	True	True
False	False	True

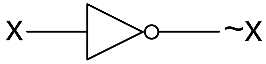
EQUIVALENCE		
A	**B**	$A \Leftrightarrow B$
True	True	True
True	False	False
False	True	False
False	False	True

NOT	
A	$\sim A$
True	False
False	True

Logic Circuits and Symbols

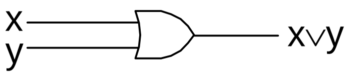
A *logic circuit* is sort of like an electrical diagram, which you can follow along by hand from one side to the other. There are three main symbols you need to know. They are often called 'gates'.

NOT symbol

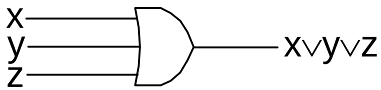


This looks like a triangle with a small circle at the tip. When you put a statement like 'x' into it, you come out with the opposite of that statement. So true becomes false, and false becomes true. Notice that there's only one input.

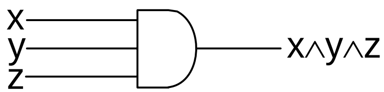
OR symbol



This is like a triangle but with *all* the sides curved. Also notice how it has *two* inputs. You can actually have more than two inputs as well, for instance, if you have x, y and z as inputs then it would look like this:



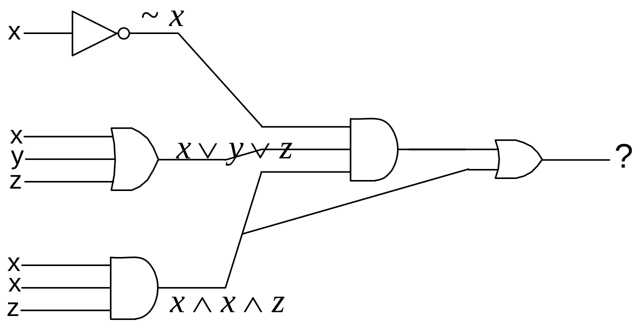
AND symbol



The AND symbol is like a bit of a rectangle with a semicircle attached to one end. It can also have more than 2 inputs, like in this particular case.

Analysing Logic Circuits

Work your way bit by bit through each part of the circuit:



Use letters first – keep 'x', 'y', and 'z' in the calculations.

Use brackets to keep each of the inputs separate in the expression for the output for the rightmost AND gate:

$$(\sim x) \wedge (x \vee y \vee z) \wedge (x \wedge x \wedge z)$$

Work through until you get a final expression:

$$[(\sim x) \wedge (x \vee y \vee z) \wedge (x \wedge x \wedge z)] \vee [x \wedge x \wedge z]$$

Evaluate by substituting in the values:

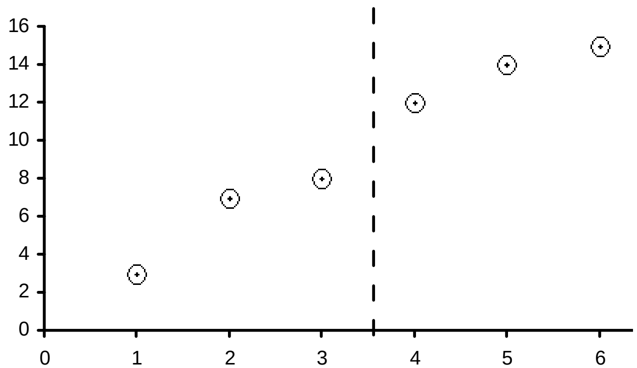
x = true, y = false, z = true

$$=> [\mathit{false} \wedge \mathit{true} \wedge \mathit{true}] \vee [\mathit{true}]$$
$$= [\mathit{false}] \vee [\mathit{true}]$$
$$= \mathit{true}$$

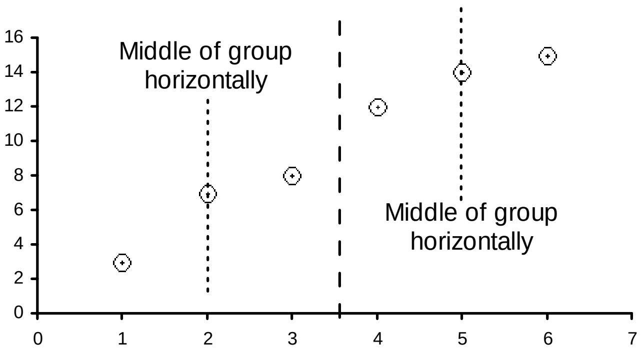
Lines of Best Fit

Visually Finding a Line of Best Fit

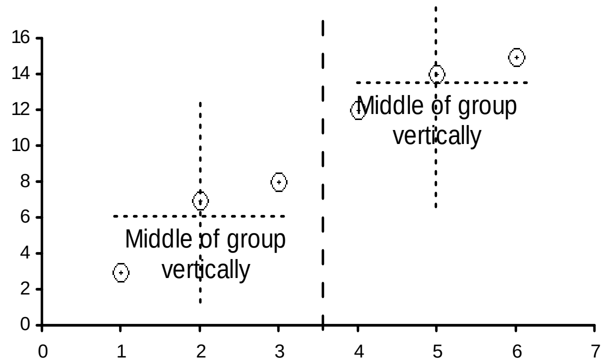
- Draw a vertical line that splits the points up into two equal sized groups. If there are an odd number of points (for instance 5), just split the groups slightly unevenly (3 in one, 2 in the other).



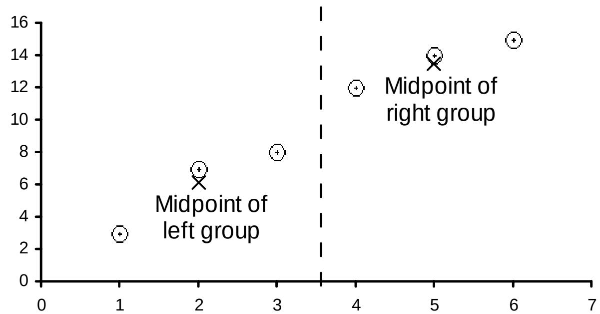
- Find the middle of each group in the horizontal direction.



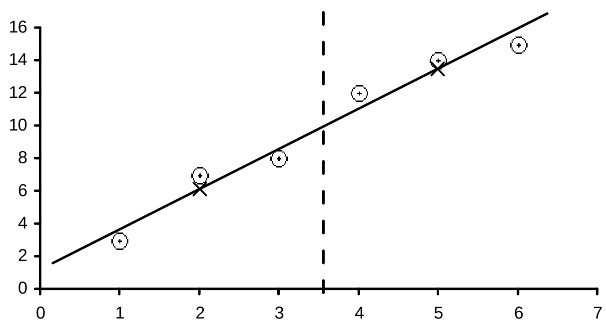
- Find the middle of each group in the vertical direction.



- Draw a cross or marker at the midpoint of each of the two groups. The midpoint is the location found in steps 2 and 3.

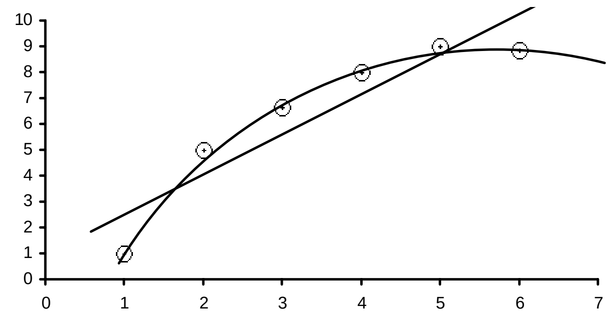


- Draw a line between these two midpoints.



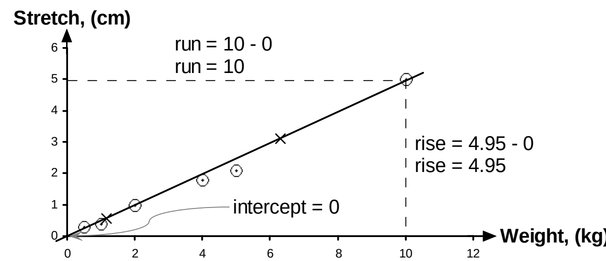
Curved Lines

Sometimes you're better off using a *curved* line to represent the data. For this data:



Notice how the curved line seems to fit the data points a lot better than the straight line.

Getting a Line Equation



Normally straight line is $y = mx + c$. But instead of 'x' we have the weight, 'w'. Instead of 'y' we have the stretch length, we'll call 's':

$$s = mw + c$$

The intercept, 'c', is going to equal '0'

$$m = \frac{rise}{run}$$

$$m = \frac{4.95}{10}$$

$$m = 0.495 \frac{cm}{kg}$$

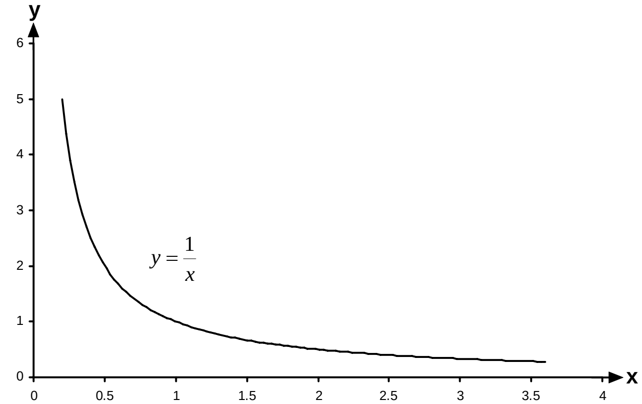
So:

$$s = 0.495w + 0$$

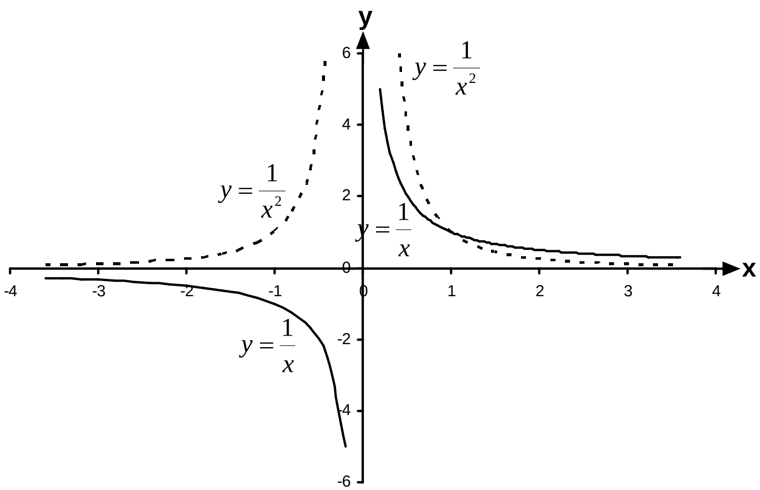
$$s = 0.495w$$

Inverse Relationships

Inverse relationships are all about one thing doing the opposite of the other. When one variable gets bigger, the other gets smaller, and vice versa. Here's the graph of a typical inverse relationship between x and y:



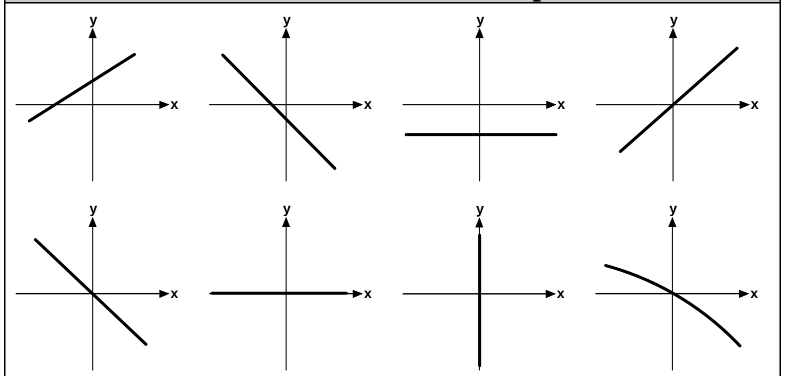
You can get inverse square $\left(y \propto \frac{1}{x^2}\right)$ or inverse cubic $\left(y \propto \frac{1}{x^3}\right)$ relationships too.



Spotting Them In Graphs

- In the area of the graph to the *right* of the y-axis, does one variable increase as the other variable decreases? The area of the graph to the *left* of the y-axis is a bit tricky – for $y = \frac{1}{x}$, as x decreases (goes more negative), y gets larger (becomes less negative). However, for $y = \frac{1}{x^2}$, as x decreases, y decreases too.
- Is the graph *undefined* at x = 0 (where the y-axis usually is)? This should happen regardless of what type of inverse relationship it is, because there's a divide by 'x' or divide by 'x^2' or something similar. Since x is zero, dividing by '0' gets you an undefined number. The value of 'y' near x = 0 should be either a very large positive number or a very large negative number.

Not inverse relationships



Spotting Them In Tables

If there is a power 1 inverse relationship $\left(y \propto \frac{1}{x}\right)$, then the product xy will be constant.

If there is a power 2 inverse relationship $\left(y \propto \frac{1}{x^2}\right)$, then the product x^2y will be constant.

x	1	2	4	8
y	64	16	4	1
xy	64	32	16	8
x^2y	64	64	64	64

The xy row doesn't have constant values along it, but the x^2y row does. This means that the relationship is an inverse squared relationship, y is inversely proportional to x^2:

$$y \propto \frac{1}{x^2}$$

Joint Variation

The most well known formula in the field of electricity is this simple one:

$$V = IR$$

Now, neither 'V', 'I' or 'R' are constants, they are all *variables* which can change their value. We would say that V varies directly as 'I' and directly as 'R'.

We could of course rearrange this equation to make 'R' the subject for instance:

$$R = \frac{V}{I}$$

Integration can be thought of as the reverse of differentiation. It is like doing differentiation in reverse.

The constant of integration

Whenever you integrate something, you always have something called the constant of integration in your answer. This is a letter, usually a 'c', that represents a number.

Integration Rules

1. If I integrate ax^n, I get $\frac{a}{n+1}x^{n+1}+c$

For example:

Function	Integrated
$4x^3$	$x^4 + c$
5	5x + c
$–3x^{-2}$	$3x^{-1} + c$

2. Rule 1 holds for just about everything except when you have something to the power '–1', i.e. 'x^{-1}', or '$1/x$' (these are the same thing). In a special case like this, the integral of '$1/x$' is '$\ln x + c$'. '$\ln$' is the natural log.

3. The integral of e^x is $e^x + c$.

4. Integration can be done in bits. Say I have $6x^2+5x^4$ and I want to integrate it. I can integrate each term separately, then add the results:

Integration

First integrate $6x^2$ to get $2x^3+a$.

Then integrate $5x^4$ to get x^5+b.

Note I have used a different letter to 'c' for the *constant of integration*. Also note that I have used a different letter in each one – they do not necessarily have the same constant of integration value.

Add the results together:

$$=> 2x^3+a+x^5+b$$
$$= x^5+2x^3+(a+b)$$
$$= x^5+2x^3+c$$

The 'c' in the final answer represents 'a + b'. Since 'a' is just a number, and 'b' is just another number, when they are added together they just represent yet another number. To make it easier to read, we can just call this number 'c'.

5. The integral of $\cos x$ ***is*** $\sin x + c$.

6. The integral of $\sin x$ ***is*** $-\cos x + c$ ***(note the negative sign).***

7. If you have a linear function raised to a power, you can integrate it. By linear function, I mean that there are not any squared terms or higher powers in it.

For instance:

Integrate $(3x-4)^3$

What you can do is represent the function in the brackets by a letter, say the letter 'a'. We can then rewrite what we have to integrate as a^3.

Integrate the letter normally – it becomes $\frac{1}{4}a^4+b$.

'b' is the constant of integration. Then divide it by the derivative of the function a with respect to x. *The derivative of function 'a' with respect to 'x' is calculated like this:*

$$a = 3x-4$$
$$\frac{da}{dx}=3$$

So if we divide our integral by '3' we get:

$$=> \frac{\frac{1}{4}a^4+b}{3}$$
$$= \frac{1}{12}a^4+\frac{b}{3}$$
$$= \frac{1}{12}a^4+c$$

Since 'b' is just a number, we can rewrite '$\frac{b}{3}$' as 'c' to make the equation easier to read.

The last step is to substitute in what 'a' represents:

$$\frac{1}{12}(3x-4)^4+c$$

Integration Symbol

There is a mathematical symbol that stands for the verb integrate. Say I want to integrate '2x + 3'. I can rewrite this without using words as:

$$\int(2x+3)dx$$

The integrate symbol is '$\int$'. I also need to say what variable I am integrating *with respect to*. This is what the 'dx' does – 'dx' means the change in 'x'. It tells me that the variable I'm integrating is the one which is changing – 'x'.

More Integration

Definite Integrals

$$\int (2x+3)dx$$

The mathematical expression above is called an indefinite integral. There is also a definite integral. The *definite integral* is like an *indefinite integral* except two numbers are specified between which to integrate. An example of a *definite integral* is:

$$\int_1^4 (2x+3)dx$$

The two numbers specified are 1 and 4. To evaluate what this definite integral is equal to, you do the following:

Step 1 *– Just integrate the function we have been doing – we get the answer* x^2+3x+c.

Step 2 *– Evaluate what this answer equals when we substitute the top number in (the 4):*

$$=> 4^2+(3\times 4)+c$$
$$=16+12+c$$
$$=28+c$$

Step 3 – *Evaluate what this answer equals when we substitute the bottom number in (1):*

$$=> 1^2+(3\times 1)+c$$
$$=1+3+c$$
$$=4+c$$

Step 4 *– Subtract step 3 from step 2:*

$$=> (28+c)-(4+c)$$
$$=24+c-c$$
$$=24$$

Note how the constant of integration cancels out.

Some Physical Meanings of Integration

There are three things that can describe a body's position and movement; position, velocity and acceleration.

- Position is where the body is located.
- Velocity is how fast the body is moving, and in which direction.
- Acceleration is a measure of whether the body's velocity is getting faster or slower, and how quickly (this is a simplified definition).

If you know one of these three things, you can obtain information about the other two using differentiation and integration.

- You can get velocity by *differentiating* the body's position function.
- You can get acceleration by *differentiating* the body's velocity function.

Likewise, you can use integration to go in reverse:

- You can get velocity by *integrating* the body's acceleration function.
- You can get position by *integrating* the body's velocity function.

Another way of visualising this is:

Differentiation: position $\longrightarrow$ velocity $\longrightarrow$ acceleration

Integration: acceleration $\longrightarrow$ velocity $\longrightarrow$ position

Example:

A truck's velocity is (5t) m/s, where t is the time in seconds since it started moving. After 10 seconds, how far has it moved from where it started?

We're interested in how far the truck has moved between the start time and the end time. So the numbers to put in are the time when the truck starts moving, and the time at the end. It starts at time = 0 and finishes at time = 10. Now we can write the definite integral:

$$\int_0^{10} (5t)dt$$

Note how now it is a 'dt' at the end instead of a 'dx'. In general, you write 'd_' and put whatever in the function is changing – in this case 't', in previous cases it was 'x'.

Integrating we get:

$$2.5t^2+c$$

We now have an expression for the truck's position after 't' seconds.

So now we work out what this expression equals for 10 seconds (the **top** number):

$$=> (2.5\times 10^2)+c$$
$$=250+c$$

Then we work out what this expression equals for 0 seconds (the **bottom** number):

$$(2.5\times 0^2)+c=c$$

Then we subtract the second result from the first:

$$(250+c)-c=250m$$

Index and Log Laws

Adding indices when you multiply

When you multiply together two identical numbers or pronumerals that are both raised to a power, you can combine them together by just *adding* the indices together:

$$x^a \times x^b = x^{a+b}$$

$$4^2 \times 4^3 = 4^{2+3} = 4^5$$

Subtracting indices when you divide

When you divide identical numbers or pronumerals each raised to a power, you need to *subtract* the indices from each other instead of adding them together.

$$x^a \div x^b = x^{a-b}$$

$$2^5 \div 2^3 = 2^{5-3} = 2^2$$

Raising powers to powers

When you raise a power to *another* power, you can rewrite the whole expression, combining the two indices or powers by *multiplying* them together:

$$\left(x^a\right)^b = x^{a \times b}$$

$$\left(4^2\right)^3 = \left(4^{2\times 3}\right) = 4^6$$

The zero index or exponent

When you raise *anything* to the power zero, you end up with the number 1:

$$\text{Anything}^0 = 1$$

Negative indices or exponents

When you raise a number to a negative power, it's equivalent to the *inverse* of that number raised to the positive of that power:

$$x^{-a} = \frac{1}{x^a} \qquad 2^{-1} = \frac{1}{2}$$

Indices and exponents that are fractions

The general way to convert from a fractional power to a root is like this:

$$a^{1/x} = \sqrt[x]{a} \qquad 5^{1/2} = \sqrt{5} \qquad 5^{1/4} = \sqrt[4]{5}$$

Fractional indices with numerators larger than one

$$2^{2/3} = 2^{1/3+1/3}$$

$$2^{2/3} = 2^{1/3} \times 2^{1/3}$$

$$2^{2/3} = \sqrt[3]{2} \times \sqrt[3]{2}$$

$$2^{2/3} = \left(\sqrt[3]{2}\right)^2$$

Common cube roots

$$\sqrt[3]{1} = 1,\ \sqrt[3]{8} = 2,\ \sqrt[3]{27} = 3,\ \sqrt[3]{64} = 4,\ \sqrt[3]{125} = 5$$

Log Laws

If we have:

$$a^c = b$$

The log statement is:

$$\log_a b = c$$

This statement says that the *base a* to the *power c* equals *b*.

Common logarithms

These all have a base of 10. For instance:

$$\log_{10} 10 = 1,\ \log_{10} 100 = 2,\ \log_{10} 1000 = 3$$

Natural logarithms

These all have the special number 'e' as a base, which is equal to about 2.718. Natural logarithms are also expressed using ln instead of log.

$$\ln_e e = 1 \qquad \ln_e e^2 = 2$$

Logarithm Laws

1. **$\log_A BC = \log_A B + \log_A C$**

   $\log_2 6 = \log_2 3 + \log_2 2$ since $2 \times 3 = 6$

2. **$\log_A(B/C) = \log_A B - \log_A C$**

   $\log_3(3) = \log_3 6 - \log_3 2$ since $6/2 = 3$

3. **$\log_A(B)^D = D \times \log_A(B)$**

   $\log_4 16 = \log_4 4^2 = 2 \times \log_4(4)$ since $16 = 4^2$

4. **$\log_{\text{some number}}$ (same number) = 1**

   $\log_{57} 57 = 1$

Change of Base Theorem

$$\log_B C = \frac{\log_A C}{\log_A B}$$

Example: To calculate what $\log_2 17$ is:

$$B = 2 \qquad C = 17$$

$A = 10$ since calculators have base 10 on them:

$$\log_2 17 = \frac{\log_{10} 17}{\log_{10} 2}$$

$$\approx 4.09$$

Answer check: 17 is only a bit more than 16, which we know is 2 to the power 4. An answer of 4.09 makes sense.

Writing fractions

Fractions can be written in lots of different ways:

With a slanting line	3/8
With a different slanting line	$^{3}\!/_{8}$
How some calculators display fractions	3⌟8

The *general form* of a fraction is:

$$\frac{\text{Numerator}}{\text{Denominator}}$$

Proper fractions

Proper fractions are fractions where the numerator is *smaller than* the denominator.

$$\frac{1}{3},\frac{5}{27},\frac{7}{57},\frac{3523}{764442}$$

Equivalent fractions

The same fraction can be written using different numbers.

$$\frac{1}{2}=\frac{10}{20}=\frac{2}{4}$$

Improper fractions

Improper fractions have numerators that are *larger than or equal to* the denominator.

$$\frac{17}{5},\frac{4}{2},\frac{37}{22},\frac{645}{276}$$

Mixed numbers

Improper fractions can also be written as *mixed* numbers. Mixed numbers *mix* both whole numbers and fractions.

Fractions

$\Rightarrow \frac{7}{5}$ Here's the original improper fraction

$=\frac{5+2}{5}$ Let's split up the numerator

$=\frac{5}{5}+\frac{2}{5}$ Let's split it up into two separate fractions

$=1+\frac{2}{5}$ $\frac{5}{5}$ is the same as 1

$=1\frac{2}{5}$ There is no need to write the plus sign

Reciprocals

To find the reciprocal of a fraction, all you have to do is swap the numerator and the denominator.

Find the reciprocal of $\frac{7}{9}$

Swap the numerator and the denominator

$\frac{9}{7}$ is the reciprocal of $\frac{7}{9}$

When you multiply a fraction by its reciprocal you get 1.

$$\frac{7}{9}\times\frac{9}{7}=\frac{7\times9}{9\times7}=\frac{63}{63}=1$$

Simplifying fractions

To simplify a fraction, you need to find the *greatest common factor* (GCF) for the numerator and denominator. For $^{63}\!/_{98}$:

63 has factors 1, 3, 7, 9, 21

98 has factors 1, 2, 7

The largest common factor is 7. The GCF is 7.

Use the GCF to divide both the numerator and denominator of the fraction:

$$\Rightarrow \frac{63}{98}\div\frac{7}{7}$$

$$=\frac{9}{14}$$

Talking About Fractions

Fraction in number form	Word form 1	Word form 2	Percentage form	Decimal form
1/2	One half	One on two	50 percent	0.5
1/3	One third	One on three	33 percent	0.33
1/4	One quarter	One on four	25 percent	0.25
1/5	One fifth	One on five	20 percent	0.20
1/6	One sixth	One on six	17 percent	0.17

Fractions as percentages

- Divide 100% by the denominator
- Multiply that by the numerator

$$\frac{3}{8}:\ 100\%\div8\times3=37.5\%$$

Addition with fractions

To add together two fractions, you have to make sure that they have a *common* denominator.

Example: Add together $\frac{4}{7}$ and $\frac{17}{21}$

First find *lowest common multiple* (LCM) of denominators:

7: 7, 14, 21, 28, 35, 42, 49, 56, 63, 70 …

21: 21, 42, 63, 84, 105

LCM is 21. Rewrite both fractions with 21 as denominator (2^{nd} fraction done already):

$$\frac{4}{7}\times\frac{3}{3}=\frac{12}{21}$$

Now do calculation:

$$\Rightarrow \frac{4}{7}+\frac{17}{21}$$
$$=\frac{12}{21}+\frac{17}{21}$$
$$=\frac{12+17}{21}$$
$$=\frac{29}{21}$$
$$=1\frac{8}{21}$$

Subtraction with fractions

Same process as for addition.

Multiplication with fractions

1. Multiply the tops of the fraction, in other words the numerators.

2. Multiply the bottoms of the fractions, in other words the denominators.

More Fractions and Decimals

3. Simplify the fraction you get from the multiplication.

$$\frac{4}{7}\times\frac{9}{5}=\frac{4\times 9}{7\times 5}=\frac{36}{35}$$

In improper fraction form this is:

$$\frac{36}{35}=1\frac{1}{35}$$

Dividing with fractions

Same as for multiplication but with one extra step at the beginning:

Step 0: Swap the numerator (top) and denominator (bottom) of the fraction doing the dividing, and change it into a multiplication.

Example: $\frac{12}{5}\div\frac{3}{2}$

Step 0: Swap the 3 and 2 and change it to a multiply:

$$\frac{12}{5}\div\frac{3}{2}=\frac{12}{5}\times\frac{2}{3}$$
$$=\frac{24}{15}=\frac{8}{5}$$
$$=1\frac{3}{5}$$

Decimal Numbers

All decimal numbers have a decimal point in them which separates the part of the number which is larger than one from the part that is smaller than one.

215.678 = 200.0 + 10.0 + 5.0 + 0.6 + 0.07 + 0.08

Rounding to a number of decimal places

Write the number down, but only to one less than the asked for number of decimal places.

Now find the ‘special’ digit. The ‘special’ digit is the digit located one more decimal place to the right than the number of decimal places you’ve been asked for. So say you’ve been asked for 3 decimal places, look at the value of the digit in the *4th* decimal place – this is the ‘special’ digit.

If the ‘special’ digit is 0, 1, 2, 3, or 4, then just write your last decimal place in as it is.

If the ‘special’ digit is a 5 or larger, you need to add one to your last decimal place.

0.777777777 to 3 decimal places becomes 0.778

Rounding to a number of significant figures

A digit is counted as a significant digit *except* in the case of zeros directly after a decimal point for a number that is smaller than 1. For instance, look at the following numbers:

427 has 3 significant digits, 0.5234 has 4 significant digits, 0.000023 only has 2 significant digits.

Rounding is same process as for decimal rounding, but start from the leftmost significant figure.

1652 to two significant figures:

- The special digit is a ‘5’. Because 5 is larger than 4, we need to add one to the last significant digit – the 6 becomes 7.
- Answer is 1.7×10^3. We don’t write 1700, since that tells the reader it is accurate to 4 significant figures.

Finance

Simple Interest

Principal: Note this is the *principal*, **not** the *principle* (which is a rule or law). The principal is the amount of money that has been lent or borrowed.

Interest rate: The interest rate tells you what percentage of the amount borrowed (the principal) needs to be paid at regular time intervals. Often the time interval is a year – so you pay back interest once a year.

Term length: The term length is how long the person who has borrowed the money has to pay interest for.

Per annum: This just means, "per year."

The simple interest formula

You use this to work out how much interest needs to be payed, if you know the principal, the interest rate, and the term length.

Here's the formula:

$$\text{simple interest paid} = \frac{PRT}{100}$$

P = principal, R = % rate, T = term length

Only one important thing to remember with this formula. The units of time you used for the *interest rate* and the *term length* have to be the same. Say I have an interest rate of 5% every *six months*. I would have to write down the term length in terms of blocks of six months. So if the term length was one year, I'd write it down in the formula as '2' – since there are two six month blocks in one year.

Example

Jason has borrowed $4000 from the bank to help pay for a new car. The bank loaned him the $4000 at an interest rate of 7% per annum, with an 18 month term. How much interest will he have to pay back to the bank over the 18 months?

How many years is 18 months? It's 1.5 years. So our term length is 1.5 years:

$$\text{simple interest paid} = \frac{\$4000 \times 7 \times 1.5}{100}$$
$$\text{simple interest paid} = \$420$$

Compound Interest

$$A_{total} = P(1+i)^n$$

A_{total} = final amount at end including principal, i = interest rate, n = number of 'rests' - the times when you stop to calculate interest.

Growth Factor: The growth factor is the *ratio* of the *final amount* to the *principal*.

Example

Sally puts $800 in her savings account at 6% yearly interest, compounding daily. How much does she have after 1 year?

Note – to get 'i' you need to divide the yearly interest rate (as a decimal) by the number of rests per year.

$$A_{total} = 800\left(1+\frac{0.06}{365}\right)^{365}$$
$$A_{total} = \$849.47$$
$$\text{Growth factor} = \frac{849.47}{800} = 1.06$$

Present and Future Values

Future values are simply how much an amount of money now will be worth in a certain number of years.

We can solve for the future value of $2000 in ten years time at 13% per annum by using the compound interest formula:

$$A = P(1+i)^n$$
$$A = 2000(1+0.13)^{10}$$
$$A = \$6789.13$$

The future value is $6789.13.

Say you want to have a certain amount of money in 10 years time. The present value of that money is the amount of money you must invest now to get that desired amount in 10 years time.

$$PV = \frac{FV}{(1+i)^n}$$

Example

I want to have $6000 in 10 years time. The current compound yearly interest rate is 5% with yearly rests. What is the present value of that $6000?

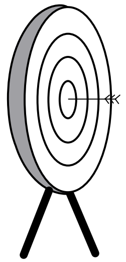
$$PV = \frac{FV}{(1+i)^n}$$
$$PV = \frac{6000}{(1+0.05)^{10}}$$
$$PV = \$3683.48$$

So if I want to have $6000 in 10 years time, I must invest $3683.48 now – this is the present value of $6000 in 10 years time.

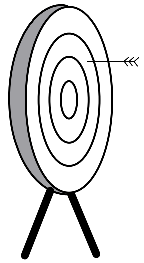
Errors and Measurement

Accuracy and Precision

Accuracy is all about how close to what you're aiming for you get. For instance, in archery, you're trying to get as close to a bullseye as possible (an arrow hitting the circular target exactly in the centre).

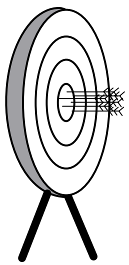


A very accurate shot

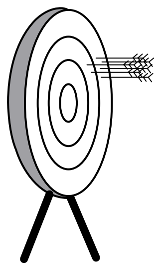


A not so accurate shot

Precision on the other hand is all about how close *repeated* shots are to each other. Precision has nothing to do however with how close you are to your desired goal.



Precision *and* accuracy



Precision but bad accuracy

Precision in Numbers

300 m - since two zeroes have been written down in the '10's and '1's part of the number, I would read this number as being 300 metres with a precision of 1 metre. If I wanted to tell the reader the measurement was 300 metres with a precision of *100 metres*, I would write:

$$3\times10^{2}\,m$$

Error

Whenever you measure something, you can't measure it exactly, there's always some amount of *error* in your measurement. The error is just the difference between what you measure and what it should be exactly:

Measurement error = measured value – correct value

So say I measured something to be 299.8 metres long, and it was actually 300 metres long:

$$\text{Error} = 299.8\text{ m} - 300\text{ m}$$
$$\text{Error} = -0.2\text{ m}$$

Absolute Error

The absolute error is just the absolute value of the error. So in the last calculation, the error was *negative* 0.2 metres. The absolute error is just the positive version of this:

$$\text{Absolute Error} = |-0.2\text{ m}|$$
$$\text{Absolute Error} = 0.2\text{ m}$$

The two vertical lines either side of the '0.2 m' mean 'take the absolute of'.

Greatest possible error

The largest error you can make doing a measurement is exactly one half of the precision of that measurement.

$$\text{Greatest Possible Error} = \frac{1}{2}\text{ of the precision}$$

For 300 metres to a precision of 1 metre:

$$\text{Value} = 300\text{ m} \pm 0.5\text{ m}$$

Relative Error

The relative error gives you an idea of how large the error is, given how big the thing you're measuring is. To calculate it divide the error you have by the true value of the thing you're measuring.

$$\text{Relative error} = \frac{\text{Error}}{\text{True value}}$$

Percentage Error

$$\text{Percentage error} = \text{Relative error} \times \frac{100}{1}\%$$

Adding and subtracting numbers with errors

Just *add* the errors (you add the errors *regardless* of whether the numbers are being added or subtracted):

$$(4.0\text{ m} \pm 0.01\text{ m}) + (1.00\text{ m} \pm 0.005\text{ m})$$
$$= 5\text{ m} \pm 0.015\text{ m}$$

Multiplication or division by an exact number

Just multiply/divide both the number *and* the error by the exact number. For instance:

$$\Rightarrow (4 \pm 0.5) \times 5$$
$$= 20 \pm 2.5$$

Multiplication of numbers with small errors

$$\text{Error} \approx \text{first number} \times \text{error in second number} + \text{second number} \times \text{error in first number}$$

$$(4 \pm 0.05) \times (7 \pm 0.1) = 28 \pm 0.75$$

Division of numbers with small errors

The **relative** error in the result of a division is the **relative** error in the numerator plus the **relative** error in the denominator.

Differentiation

Differentiation Rules

1. For any x^b, its derivative is simply $b \times x^{b-1}$.

The derivative of x^5 is $5x^4$.

2. For any ax^b its derivative is $b \times ax^{b-1}$. The power is put out in front of the expression, and then the power is reduced by 1. Remember anything to the power 1 is just itself, and anything to the power 0 is 1.

The derivative of $3x^4$ is $12x^3$.

The derivative of $3x^{-4}$ is $-12x^{-5}$

3. When you have terms separated by '+'s or '–'s you can find the derivative of each term then add them together to find the derivative of the whole function.

The derivative of $2x^4 + 3x^2 - 3x + 2$ is $8x^3 + 6x - 3$

Note that the 2 just disappears. All constants (numbers by themselves without algebraic symbols) disappear when you find the derivative.

4. The derivative of sin x is cos x

5. The derivative of cos x is –sin x (note the change in sign).

6. The derivative of e^x is e^x.

7. The derivative of ln x is 1/x, for x > 0. There is no ln x when x ≤ 0 (ln is the natural logarithm)

Note that the derivative can be written a few different ways:

- If you have $f(x) = \ldots$, then the derivative can be written $f'(x) = \ldots$
- If you have $y = \ldots$, then the derivative can be written $y' = \ldots$
- If you have a function with 'x' as the variable being differentiated, the derivative can be written as:

$$\frac{d}{dx}(something)$$

'Something' is the function which has 'x' in it.

Chain Rule

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

In words, this says the derivative of y with respect to x is equal to the derivative of y with respect to u times the derivative of u with respect to x. Note if you cancel out the 'du's on the right hand side, you are left with the left hand side.

$$4(3x+4)^5$$

$$\frac{dy}{du} = 20u^4 \text{ and } \frac{du}{dx} = 3$$

So we can use our differentiation rules to calculate that

$$\frac{dy}{dx} = 20u^4 \times 3 = 60u^4.$$

We can then substitute in for what u is and give the answer as

$$60(3x + 4)^4$$

Product Rule

$$y = (3x^2 + 2x)(2x^4 - 5)$$

The product rule simply states that if the function can be split up into a sub-function multiplied by another sub-function, then:

$$y' = u'v + uv' \quad \text{or} \quad \frac{dy}{dx} = \frac{du}{dx}v + u\frac{dv}{dx}$$

Let u = $3x^2 + 2x$ which means that $u' = 6x + 2$

Let v = $2x^4 - 5$ which means that $v' = 8x^3$

$$\frac{dy}{dx} = (6x+2)(2x^4 - 5) + (3x^2 + 2x)8x^3$$

$$= 36x^5 + 20x^4 - 30x - 10$$

More Differentiation

Quotient Rule

This rule is for when you can split your function into one sub-function divided by another sub-function. If the two sub- functions are u and v:

$$y = \frac{u}{v}$$

Then:

$$y' = \frac{u'v - uv'}{v^2}$$

Example:

Find the derivative of $y = \frac{5x}{(6x^2 + 2x)}$

Firstly:

Let u = 5x

Let v = $6x^2$ + 2x.

Which gives:

$$u' = 5$$

$$v' = 12x + 2$$

Then:

$$y' = \frac{5 \times (6x^2 + 2x) - 5x \times (12x + 2)}{\left(6x^2 + 2x\right)^2}$$

$$= \frac{30x^2 + 10x - 60x^2 - 10x}{\left(6x^2 + 2x\right)\left(6x^2 + 2x\right)}$$

$$= \frac{-30x^2}{x^2\left(6x + 2\right)\left(6x + 2\right)}$$

$$= \frac{-30}{\left(6x + 2\right)^2}$$

Handy Hint - Writing letters in calculus

Be careful when you write ‘u’ and ‘v’ – it is easy to mix them up! You could use the letters ‘a’ and ‘b’ just as easily, ‘u’ and ‘v’ are used here because many textbooks use them.

Handy Hint - Starting the question

When you get a derivative question, your first decision should be which rule to use. You can then write your ‘u’s etc. out so they are easy to use:

$$u = \ldots$$

$$u' = \ldots$$

$$v = \ldots$$

$$v' = \ldots$$

Handy Hint - Product or quotient rule

Any quotient rule problem can be turned into a product rule problem. For example:

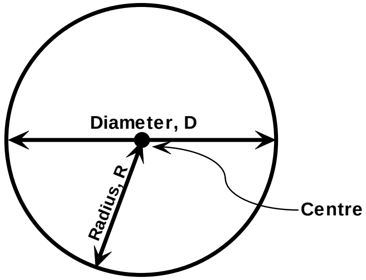
$$y = \frac{5x}{(6x^2 + 2x)}$$

$$= (5x) \times \left(6x^2 + 2x\right)^{-1}$$

$\left(6x^2 + 2x\right)$ is really $\left(6x^2 + 2x\right)^1$. When we change it from dividing to multiplying, we change its power from 1 to –1. In effect we are moving it from the denominator to the numerator – the sign of the power always changes when we do this.

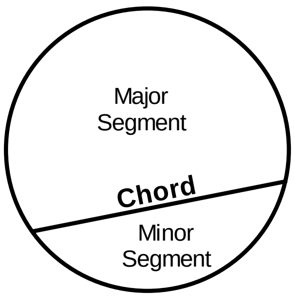
Circles

Circle



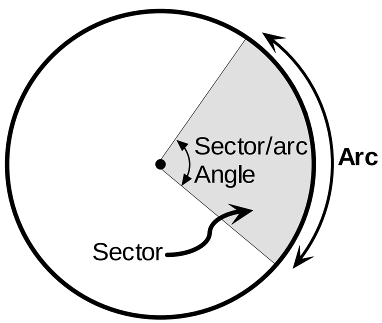
Chords and segments

A *chord* is a straight line that joins any two points on the boundary of a circle. When you draw a chord, you divide the circle into two areas. These areas are called *segments*. The smaller area is called a *minor segment*. The larger area is called a *major segment*.



Arcs

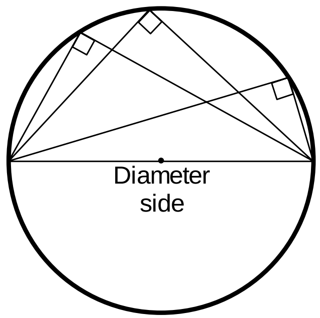
An *arc* is the part of the circle's circumference taken up by the sector:



$$\text{Arc length} = \frac{\text{Arc angle}}{360°} \times 2\pi R$$

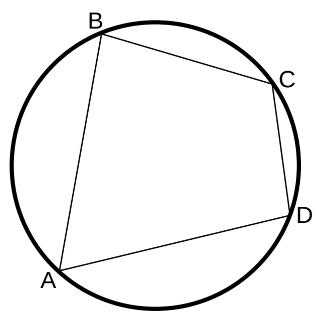
Triangles inside circles

A triangle drawn inside a circle with one side as the diameter of the circle and all corners touching the circumference has a special *property*. The angle in the triangle opposite the diameter side is always 90°:



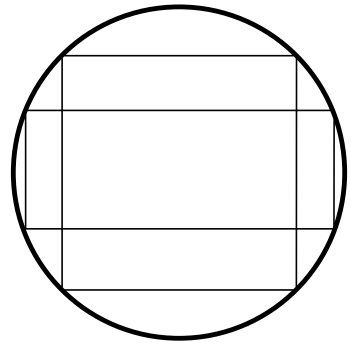
Cyclic quadrilaterals

You can fit four-sided shapes inside a circle – quadrilaterals. Not all quadrilaterals will have *all four* corners touching the circumference of the circle. Quadrilaterals that do however are known as *cyclic quadrilaterals*. Opposite angles in cyclic quadrilaterals are *supplementary* (add to 180°).

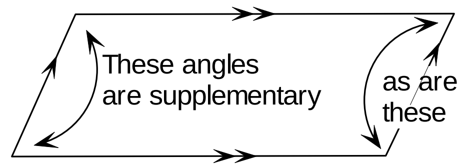


If both sets of opposite angles in a quadrilateral are supplementary, then you can draw a single circle that goes through all four corners of the quadrilateral. If they aren't supplementary, then you can't draw a single circle through all four corners.

Rectangles are cyclic quadrilaterals because all the angles inside a rectangle are 90°. Opposite angles obviously add up to 180° then. A square is a cyclic quadrilateral too for the same reason.



Parallelograms and trapeziums are *not* cyclic quadrilaterals because their opposite angles don't add to 180°. The only exception for parallelograms is when they are a rectangle.



Area of Circle

$$\text{Circle area} = \pi r^2$$

Circumference of Circle

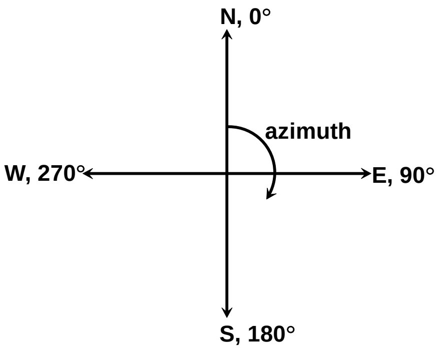
$$\text{Circumference} = 2\pi R$$

Bearings and Azimuths

Bearings are a way of talking about directions using angles. This is very useful for people in ships or planes, because you can use bearings to tell them the direction they need to travel in. There are two main ways to describe the direction.

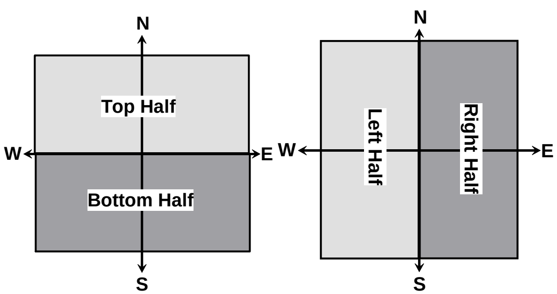
Azimuths

Azimuths describe a direction as being somewhere between 0 and 360 degrees. 0 degrees is the direction north, which is usually 'up' when you draw it on a piece of paper. The angles increase as you travel in a *clockwise* direction from north.

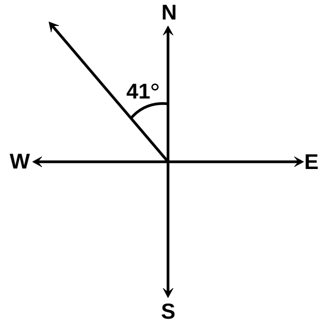


Bearings

Bearings use a combination of angles and compass directions to describe a direction. A direction is described by how far towards east or west it is from north or south.



Example



It's obviously in the top half, so we'll be describing how *far away from north* it is. So we can write something like:

Bearing is … degrees …. of north

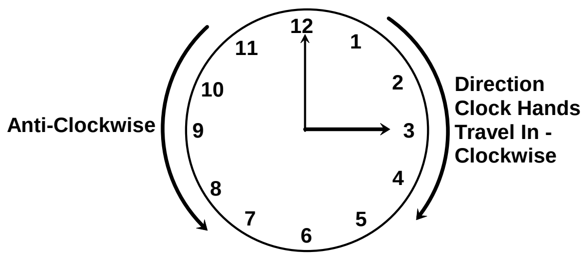
It's in the left half, so we'll be using west:

Bearing is … degrees west of north

So now all we've got to do is provide the number of degrees. Well, if we start at north, and rotate towards west until we reach our direction, how many degrees have we rotated? Luckily, it's already marked on the diagram - 41°. So we can write:

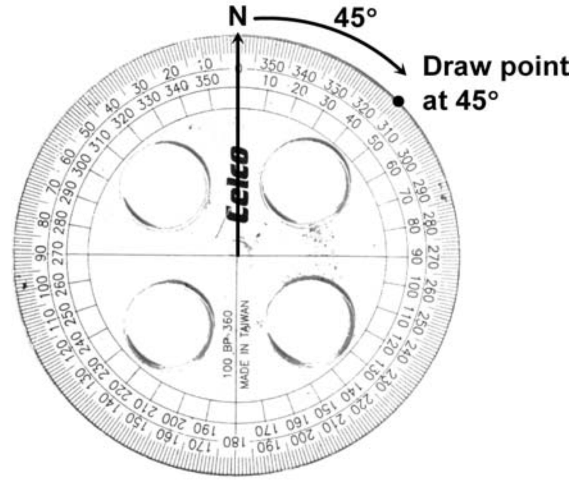
Bearing is 41° degrees west of north

Clockwise and anti-clockwise

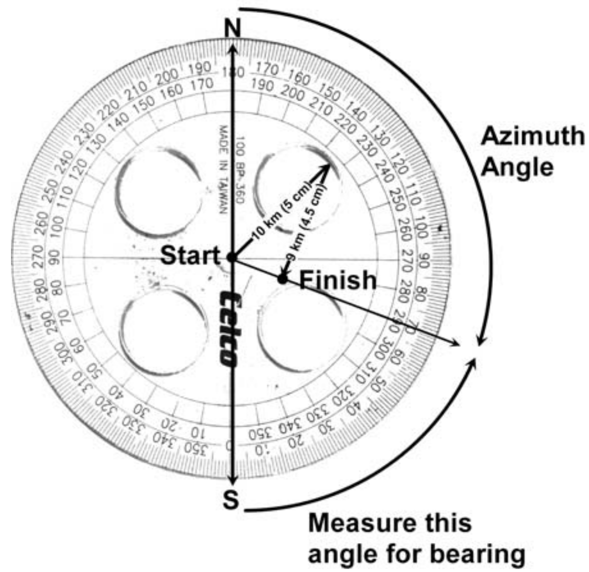


Using Your Protractor

Measuring 45° east of north:



Measuring both an azimuth angle and a bearing from the 'Start' location to the 'Finish' location:



Azimuth = 110°.

Bearing = 70 degrees east of south.

Algebra Introduction

Variables or Pronumerals

The first and biggest thing to understand is what a *variable* is in algebra. Say I have the following equation:

$$x + 7 = 9$$

If you read this new equation out aloud, you should say something like: "x plus seven equals nine". 'x' is what we call a *variable*, or *pronumeral*. Variables are used to *represent* numbers.

Coefficients of variables

A coefficient is the number in front of a variable. Look at the following expression:

$$5x$$

The number in front of the 'x' is its coefficient: 5. What about:

$$-4x + 3y$$

The coefficient of 'x' is '–4' and the coefficient of y is '3'.

Addition and Subtraction

$$3x + 4x$$

I can read this equation as "Three lots of x plus four lots of x". What happens when I have 3 of something, and I add another 4 of the same thing? Simple – I end up with 7 of that thing. So in this case, I can rewrite this expression:

$$\Rightarrow 3x + 4x$$
$$= 7x$$

What about if I have something like:

$$6x + 3y$$

Can I add these two together? The answer is NO. I can only add together variables that are the same. 'x' and 'y' are *different* variables, so I can't add them together.

Multiplying and dividing variables

Say I wanted to multiply 'x' by 'y'. I'd write this as:

$$xy$$

When you're multiplying variables together, often the multiplication symbol isn't written. The last expression is exactly the same as:

$$x \times y$$

except it doesn't have the multiplication symbol.

You can also divide variables by other variables:

$$\frac{x}{y} \text{ or } {}^{x}\!/\!_{y} \text{ or } x/y \text{ or } x \div y$$

Expressions, equations and terms

A term is part of an equation which has one or more variables or pronumerals in it, but no '+' signs or '–' signs in it. For instance:

- $\frac{42x^2y^3}{\sqrt{43xy}}$ is also a *term* because it has the two pronumerals *x* and *y* in it, but no '+' or '–' signs in it.
- $(4x + 3y)$ is *not* a term because it has a '+' sign in it. In fact, it is made up of *two* terms, '4x' and '3y'.

An *expression* is a group of terms connected by '+' or '–' signs. There can be as many terms as you want, as long as there is at least 1. For instance,

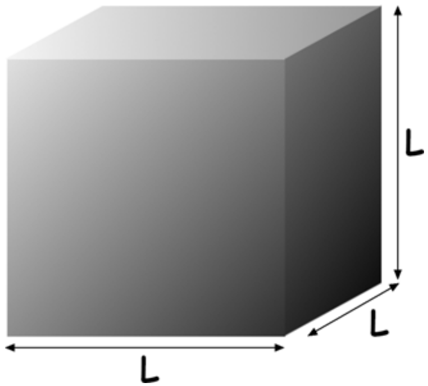
$$3x^2 + 2x - 15.5$$

is an expression, because it contains three terms, which are joined together by a '+' and a '–' sign.

An *equation* is made up of two expressions with an '=' sign between them. One expression is on the left of the '=' sign, and the other expression is on the right of the '=' sign. Here's one example of an equation:

$$3x + 4 = 5x - 2$$

3D Solids

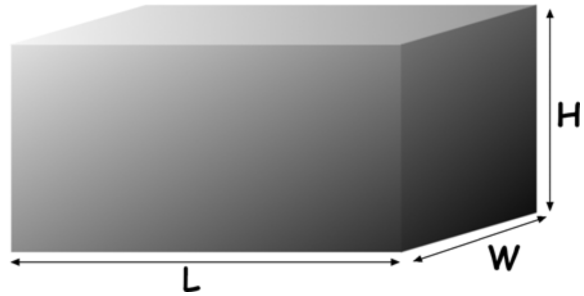


CUBE

$V_{cube} = L^3$

$SA_{cube} = 6L^2$

6 surfaces, 6 faces
12 edges, 8 vertices



RECTANGULAR PRISM

$V_{rect.\,prism} = L \times H \times W$

$SA_{rect.\,prism} = 2(LW + LH + HW)$

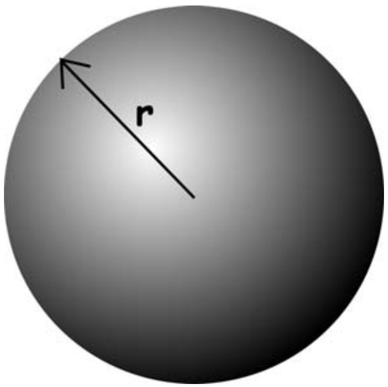
6 surfaces, 6 faces
12 edges, 8 vertices

SPHERE

$V_{sphere} = \frac{4}{3}\pi r^3$

$SA_{sphere} = 4\pi r^2$

1 surface, 0 faces
0 edges, 0 vertices

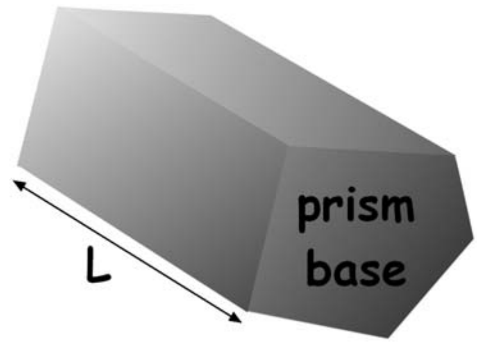


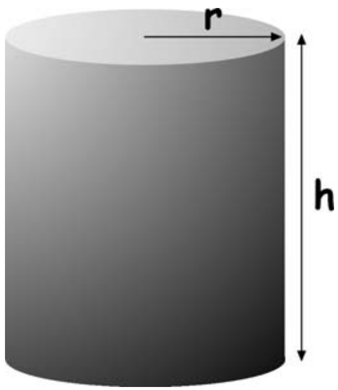
GENERAL PRISM

$V_{prism} = L \times \text{base area}$

$SA_{prism} = 2 \times \text{base area} + \text{L} \times \text{base perimeter}$

Surfaces: 2 + number of base sides
Faces: 2 + number of base sides
Edges: 3 × number of base sides
Vertices: 2 × number of base sides



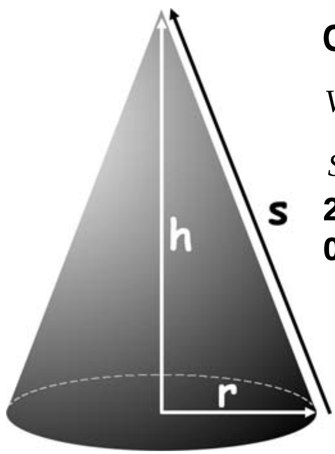


CYLINDER

$V_{cylinder} = \pi r^2 h$

$SA_{cylinder} = 2\pi r(r + h)$

3 surfaces, 2 faces
0 edges, 0 vertices



CONE

$V_{cone} = \frac{1}{3}\pi r^2 h$

$SA_{cone} = \pi r(r + s)$

2 surfaces, 1 face
0 edges, 1 vertex

PYRAMID

$V_{pyramid} = \frac{1}{3} \times \text{base area} \times \text{height}$

$SA_{pyramid} = \text{base area} + \text{triangular side areas}$

Surfaces: 1 + number of base sides
Faces: 1 + number of base sides
Edges: 2 × number of base sides
Vertices: 1 + number of base sides