Optimization of Copper to Iron Ratio in Work Solenoids

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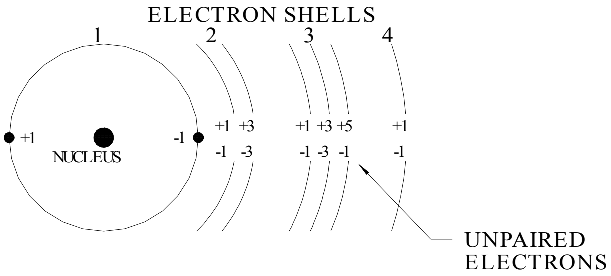
Introduction – Work solenoids (as opposed to the fundamental form of a solenoid; i.e., a simple inductor containing little or no iron components) have the purpose of producing significant force or work on another object to move it against an opposing force or to accelerate it within a required time period. An iron based core and armature (plunger) greatly increases the force potential otherwise not realized strictly from a copper coil alone and therefore are principal components of a work solenoid. This effect is due to induction. Induction in iron (and a few other elements) is possible due to the fact that the iron atom has an imbalance of electron spins; specifically the third electron shell has an imbalance of four unpaired electrons which gives the atom a net magnetic moment. [1]



Fit. 1 Iron Atom Electrons

The iron atoms tend to magnetically align in series and parallel forming small clusters called domains; however, the domains randomly align relative to each other and thus give the iron a neutral magnetic moment and the iron is said to be demagnetized (at least from the measuring ability of an external observer). That is, until the solenoid coil's magnetic field induces the domains to align according to the coil's polarity and field strength. A small coil field begins to align domains which influence neighboring domains and the field is said to propagate along the iron path. Domain switching has a finite time element which contributes, along with an increasing inductance, to the relatively slow logarithmic current rise in the iron core inductor. Since the iron volume has a finite number of domains, and with an increase in coil mmf (the coil's ampere-turns) the internal flux reaches a level where added coil current produces a diminishing rate of domain alignments and an eventual saturation of the iron. This reduction begins at the observed "knee" followed by a leveling off of flux density (B) as recorded on a BH curve. The iron path within this saturating region begins to create significant losses (mmf drops, analogous to voltage drops across a resistor) that reduces the mmf across the working air gap where it is needed to provide the potential energy for performing work. The relationship of B and H is highly non-linear across the usable range of flux densities and, for computer computations, requires a polynomial to define the relationship. Early work required a tedious referral to a table or curve of the appropriate steel alloy. For this discussion, a typical d.c. solenoid alloy would be no. 1008, or 1215, or 12L14 which can be defined by the equation:

$$H = a + bB + cB^2 + dB^3 + eB^4 + fB^5 + gB^6$$

where B is the flux density in webers/meter^2 and H is ampere-turns/meter and:
a = 110.529; b = -1700.2762; c = 14226.658; d = -41673.495; e = 57423.444;
f = -37223.825; g = 9235.1829.

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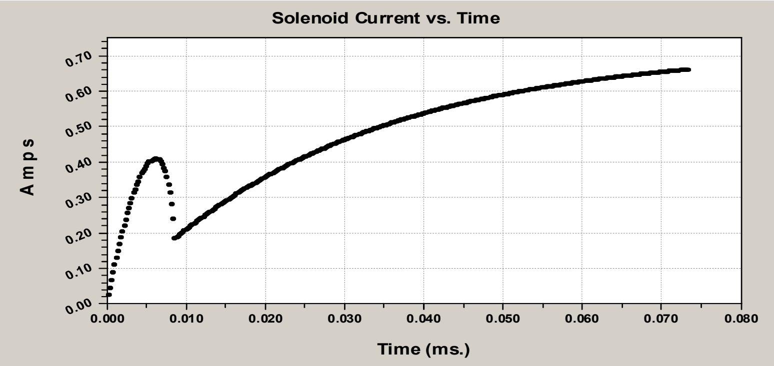
[1] Encyclopedia Britannica, vol. xiv, Magnetism, p.660

For the subject at hand, bear in mind that a work solenoid will typically have a closed series iron circuit except for the main variable (working) air gap and a requisite clearance air gap between the stator and moving plunger. The faces of the fixed stator pole and plunger pole may have any variety of shapes to "tune" the closing air gap for a preferred rate of change of permeance and resulting force profile. The question becomes, how much internal space should be allocated to copper and how much for the iron circuit(?).

The goal of the designer is to optimize a solenoid design that fits an allowable volume, produces the minimum specified force, stays within power supply limits, has the ability to dissipate its internally generated heat and takes into account (especially when acceleration time is a requirement) the mass of the moved load as well as its own armature mass. Efficiency requires the process of finding the best ratio of copper volume to iron volume for the overall allowable volume of the solenoid. This effort requires the solenoid to operate, under most circumstances, at or near the knee on the BH curve of the iron. Two fundamental factors benefit: (a) The domain alignments of the iron atoms can be viewed as an amplification factor of the coil mmf which peaks at maximum permeability (B/H). (b) Minimum required coil power is achieved resulting in a lower coil temperature rise and its attendant minimized coil resistance. Optimization of the copper and iron ratio is more productive than attempting Herculean efforts at winding a precisely perfect coil.

It should be noted that optimization of a solenoid intended for high response driving its own plunger mass or to impact a target component may differ from the one moving a spring or friction load which relies on static force. When accelerating only against the inertia of a mass the rapid acceleration will create a significant counter emf in the coil that prohibits the current, flux, and force from rising to their maximum static levels. The graph of Fig.2 shows a typical dynamic plot where the current peak during plunger movement reaches only about 57% of its steady state level.

Fig. 2 - Coil current during and following plunger movement.



During plunger acceleration, the counter emf voltage is attributed to the changing levels of current, total ampere turns in the circuit, flux, and inductance following an increment of time, dt. These factors are implicit in the equation $y = 1 - e^{-tR/L}$ where **y** is the fraction of the steady state voltage or current at the time **t**. The equivalent equation demonstrates this,

$$y = 1 - e^{\frac{-tRi^2}{\Im\phi}}$$

Upon calculating the response time and peak accelerating current (or flux density) it may be observed if further effort is advised to improve these dynamics. The designer then has the task of revising the copper and iron ratio to optimize the response time.

When optimization is required it helps to write a computer routine that incrementally trades iron (steel) volume for copper volume and computes the forces for each increment.[2] (The alternative is to analyze, via fea, incremental ratios of copper and iron requiring repeated geometry revisions). At the point of cu:fe optimization the force will peak for the fixed level of coil power and stroke position. This method also requires that the routine include a coil winding calculation or method for determining the mmf (ampere turns) of the coil volume (at the established coil power) for convergence by the summed mmf drops in the steel and air gaps. Following a well written program, solenoid analysis can be computed in seconds. The necessary steps include:

a) Define the solenoid case outside diameter and length. Define the coil bobbin wall thicknesses.
b) Set a minimum iron cross sectional path area to begin; use the same area for the case walls, plunger, stator pole and end walls. (After each step thru (k), increment the iron area which will increment the coil parameters and resulting force).
c) Calculate the case inside diameter, plunger diameter, etc. based upon area.
d) Determine the coil winding o.d., i.d., L, based upon the prior dimensions in a, b, and c. Calculate a coil to find the coil resistance and number of turns (see method below). Calculate the coil mmf based upon allowable coil watts.
e) Calculate the air gap permeances based upon geometry and stroke setting. This may be the most difficult step in the program and referral to a good magnetics reference may be required.
f) Set a beginning, arbitrary value of flux. Calculate the flux density B (based upon area in b).
g) Calculate H for the value of B from a BH equation and calculate the mmf drops for each iron component and the mmf drops across the working and clearance air gaps.
h) Sum all mmf drops and compare them to the coil mmf in d. If the summed mmf drops are greater than the coil mmf then the flux is too high. Reduce the flux by a small fixed percentage and reenter the routine at step f (and vice versa, but at a different numerical percentage, if the mmf drops are too low).When the mmf values converge to within about1% then proceed to (k).
k) Convergence of the mmf drops and coil mmf indicates the true value of flux for the geometry and power level as stated. The solenoid force is calculated based upon the flux and permeance or mmf of the working air gap. Air gap energy is $w = \phi^2/2P$ or $w = \mathscr{F}\phi/2$; and force $F = dw/ds$ where ds is a small increment of plunger movement. Alternatively, $F = B^2 A / 2\mu_0$, where B is the flux density at the main working air gap and $\mu_O = 4\pi \bullet 10^{-7}$.

Further refinements, if desired, can be carried out using fea modeling to observe areas of flux density that are undesirably high, such as in corners (add a radius), or very low for potential weight reduction. Such refinements are appropriate for aerospace applications or for cost considerations.

Thanks to several decades of solenoid manufacturing where many sizes and styles of solenoids have been produced and documented, a designer can quickly estimate the limitations of size versus power dissipation and other factors to provide a starting point for a new design. If catalog data is unavailable the solenoid power rating can be estimated by the following equations. This continuous duty power rating will allow a final stabilized coil temperature of about 105°C within an ambient air temperature of 20°C. As a side note, a case temperature of about 60°C is the threshold where the solenoid can no longer be held comfortably in your hand; 105°C will boil water and can burn you. Incidentally, the outside case temperature will be about 20° cooler than the copper coil if the coil is loose-fitted into the case; i.e., not potted or encapsulated, when at its stabilized temperature.

Allowable Coil Watts: For computing the continuous power rating (P), first calculate the total volume (v) of the solenoid (here volume is in cubic inches).

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[2] A force computational method is the subject of Macro_Magnetics.pdf.

If v=> 2.06 then P=1.213*v + 6.0 (watts).
If v=> 0.186 and v<2.06 then P=3.772*v + 0.73 (watts).
If v<0.186 then P=7.709*v (watts).

Coil Calculation Method: In 1857 J.R. Brown of Brown & Sharpe developed the magnet wire gauge sizes (awg; American Wire Gauge). Each wire size diameter can be calculated directly from the awg number by the equation $d = 0.46/92^{(awg+3)/39}$ (**d** is in inches). For awg 0000 use -3 in the exponent, etc. This is the bare copper diameter and must be modified to include the insulation layer by:

$$dI = \mathbf{d}+4.38727\text{E-03}*\mathbf{d}\hat{\ }0.343976 \quad \text{(Insulated wire diameter)}$$

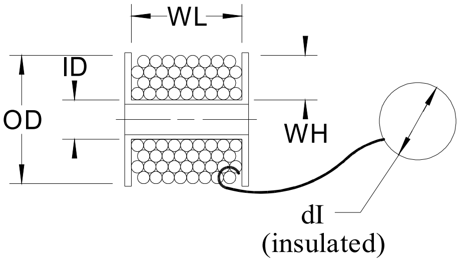

Fig. 3 Bobbin-Wound Coil

Coil winding steps after finding the insulated wire diameter, **dI**, above (units in inches); as written in pcbasic:

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 5 User must input od, id, wL, awg
10 pi=3.14159:rad=pi/180
12 d=.46/92^((awg+3)/39):di=d+4.38727E-03*d^.343976
20 wh=(od-id)/2 : xwh=di
30 tL=int(wL/di)-1: 'turns/layer
40 xwh=di : nL=1: goto 60
50 xwh=xwh+di*cos(30*rad):nL=nL+1
60 if xwh+di*cos(30*rad)<wh then goto 50
80 nt=nL*tL: 'total turns
90 R=(xwh+id)*nt*6.787388E-07/(d^2/4) : 'coil res.@20°C
100 PRINT nt ; R
110 end
```

Note that the line for resistance uses the bare copper diameter **d**, not the insulated diameter, **dI**.
Force profiles are not limited to the familiar exponential force increase during air gap closure that are characteristic of flat face and conical pole faces but can have starting forces that exceed the ending force and exhibit a more linear force curve.[3] This characteristic is not due to springs but is the result of energy distribution by means of metering the flux to the working air gap as a function of the closing gap. This subject is beyond the scope of this article and, of course, suggests fea modeling to achieve the appropriate magnetic focus at the air gap.

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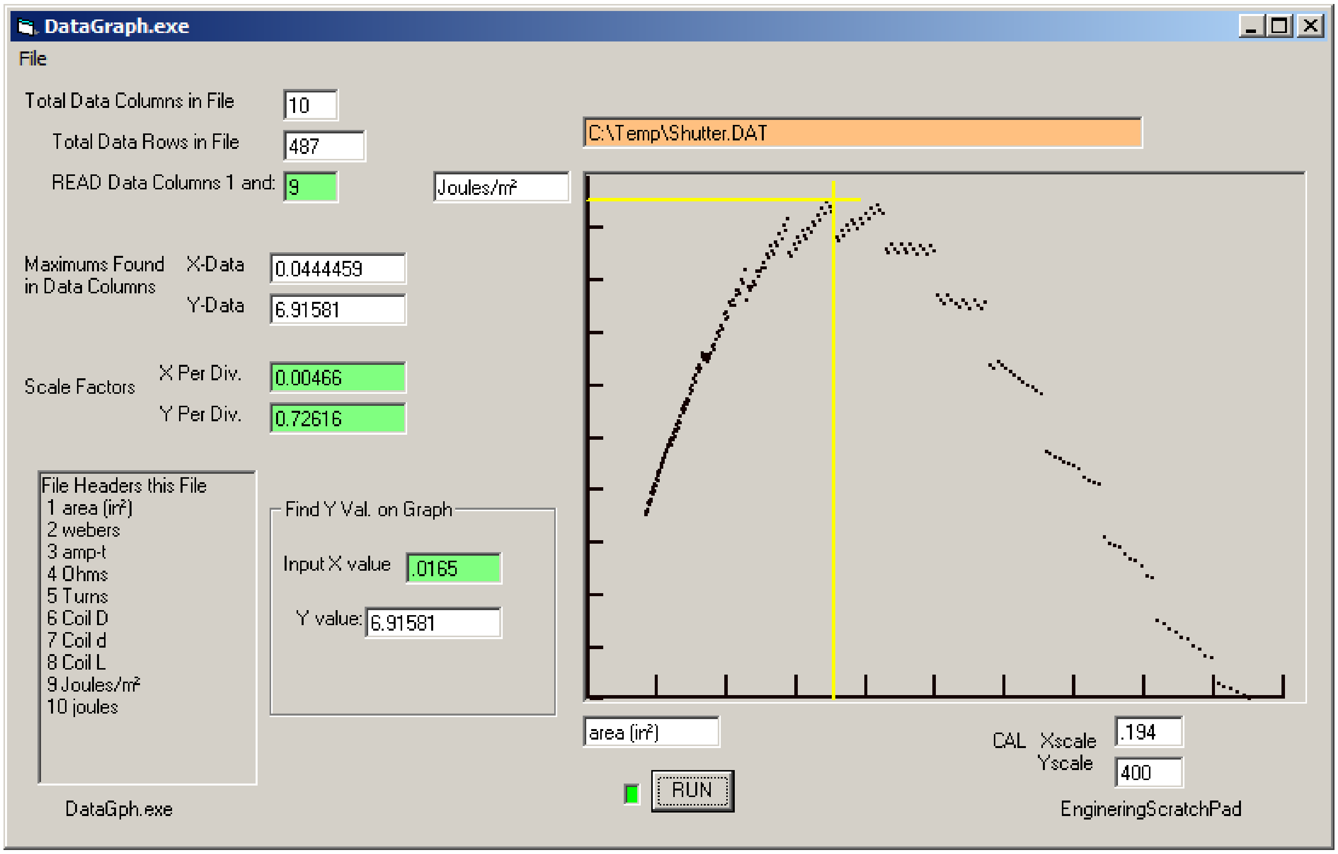
[3] The saturable pole solenoid has a higher starting force than its flat or conical pole counterpart and characterizes a linear force profile thus reducing ending shock and permitting analog positioning of the plunger. This subject is discussed in the 1986 SAE publication 860759 (Rotary and Linear DC Proportional Solenoids).

The Cu:Fe Optimization Analysis -

Figure 4 shows results of a specific solenoid copper to iron ratio optimization. This is a small rotary solenoid used to open and close a camera shutter for an IR thermal imager. Power must be minimized and speed of operation maximized. Actuation of the shutter is by powering the solenoid and de-actuation is by means of a return spring. The solenoid therefore must accelerate the shutter's moment of inertia, its gravitational torque (in certain orientations of the camera) and the torque imposed by the return spring. With a given limitation for electrical power (in this case 0.67 watts) and the solenoid's fixed diameter and length it is obvious that no intuitive guess would optimize the ratio of the iron volume and copper coil volume to achieve maximum torque of the solenoid. The rationale for optimization in the computerized analysis is to maximize the magnetic field energy per area (joules/m²) in the radial air gap beween rotor and stator poles.



Total Data Columns in File: 10
Total Data Rows in File: 487
READ Data Columns 1 and: 9
Joules/m²
C:\Temp\Shutter.DAT
Maximums Found in Data Columns X-Data: 0.0444459
Y-Data: 6.91581
Scale Factors X Per Div.: 0.00466
Y Per Div.: 0.72616

File Headers this File
1 area (in²)
2 webers
3 amp-t
4 Ohms
5 Turns
6 Coil D
7 Coil d
8 Coil L
9 Joules/m²
10 joules

Find Y Val. on Graph
Input X value: .0165
Y value: 6.91581

area (in²)
CAL Xscale: .194
Yscale: 400

Fig. 4 Graphical Data Analysis for Cu:Fe Optimization

Shown in the data graph are many iterations in which the solenoid's iron area begins at some value and increases by 1% per iteration. For each iteration, as the iron area (and volume) increases the copper coil volume must decrease to maintain a fixed diameter and length of the solenoid. The coil is recomputed for each iteration and coil power is maintained as a fixed constant. The gaps appearing in the graph are a result of the coil winding efficiency as its bobbin dimensions change.

The graph's X-axis is iron area (in²) and the Y-axis is energy in the air gap per square area (joules/meter²). The air gap energy peaks at a value of 0.016 square inches of flux path area.

After finding the peak energy, any of the File Header parameters can be selected and measured at the same area location; specifically the copper od, id, and L to design the coil for manufacture.