Solenoid Electromagnetics

Scope: This article describes the basic physics and mathematical solutions for calculating and predicting the forces generated by an electromagnet to a high degree of accuracy without the aid of finite element analysis (fea) software. This methodology was practiced up through the latter half of the 20th century (and prior) for iron core solenoids due to the absence of fea software. When computers became available to ordinary manufacturing facilities and individuals in the 1980's the burden of relating the magnetic flux density (B) in iron to its magnetization (H), but still without fea software, was greatly reduced as will be presented. Prior to the computer, "solving" was approximate due to the tedious referral to a BH graph or lookup table to obtain H and then making the equation calculations, possibly many times depending upon the skill of the analyst to estimate the next trial flux level to achieve a suitable convergence of the dependent and independent magnetomotive force levels as will be explained.

Background: James Clerk Maxwell's equations as related to electromagnetics, in the mid-1800's, were presented as partial differential equations and were prior to any significant industrial manufacturing of electromagnets. The first practical electromagnet solenoid was patented by I.A. Timmis in 1893[1]. In 1941 Dr. Alan Hazeltine, at Stevens Institute of Technology, commissioned Herbert C. Roters to author a textbook[2] for engineering students there and to simplify Maxwell's mathematical rigors into equivalent algebraic identities. This article uses these guidelines, without the exhaustive detail as found in his text, to provide a straight forward understanding of force calculations.

Solenoid anatomy: Solenoid electromagnets, also classified as work solenoids, have the purpose of producing significant force or work on another object to move it against an opposing force or to accelerate it within a required time period. An iron based core and armature (plunger) greatly increases the force potential otherwise not realized strictly from a copper coil alone and therefore are principal components of a work solenoid. This amplifying effect is due to induction. Induction in iron (and a few other elements)[3] is possible due to the fact that the iron atom (Fig.1) has an imbalance of electron spins; specifically the third electron shell has an imbalance of four unpaired electrons which gives the atom a net magnetic moment.[4]

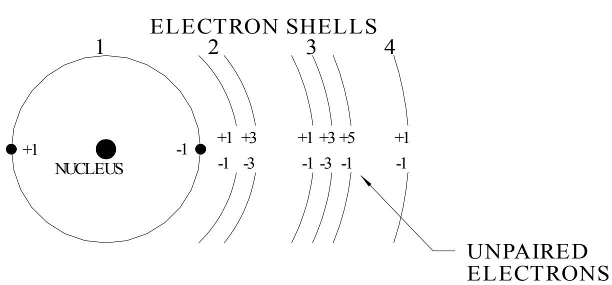

Fig.1 - Unpaired Electrons in the Iron Atom

1 U.S. patent no. 506,282

2 Electromagnetic Devices by Herbert C. Roters, 1941, published by John Wiley & Sons. References in this article are used by permission of the publisher.

3 The ferromagnetic atoms are elemental iron, cobalt, nickel, gadolinium, and dysprosium. Modern Technical Physics, Beiser, 1966.

4 Encyclopedia Britannica, vol. xiv, Magnetism, p.660.

The iron atoms tend to magnetically align in series and parallel forming small clusters called domains, however, the domains randomly align relative to each other and thus give the iron a neutral magnetic moment and the iron is said to be demagnetized (at least from the measuring ability of an external observer). That is, until the solenoid coil's magnetic field induces the domains to align according to the coil's polarity and field strength. See Fig.2 where a cylindrical work solenoid is pictured showing the iron (steel) components and air gaps. Other forms of work solenoids are common whereas the cylindrical form is usually the most electrically efficient.

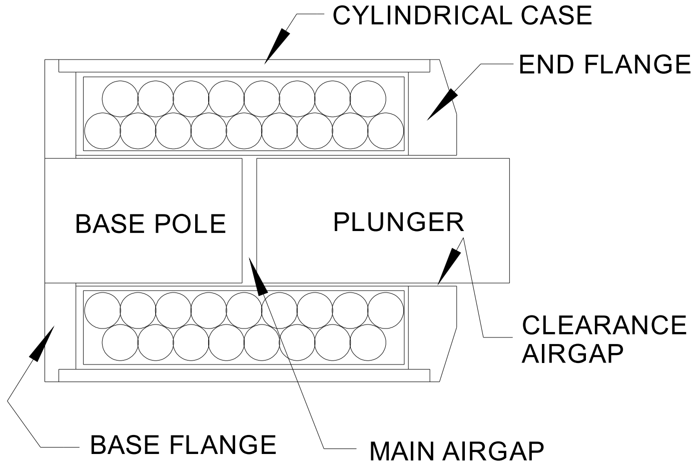

Fig.2 - Cylindrical Work Solenoid

Induction and Flux Propagation: Suppose a magnetic flux is passed (propagated) uniformly thru the ferrous case tube as shown in Fig. 3. Let the case dimensions, o.d., i.d., and length, be known as well as the type of steel. The presence of a flux within the steel causes an mmf drop (ampere-turns) across the length of the tube (one of several macro elements of the solenoid). Likewise, each steel element in the solenoid produces an mmf drop due to the same propagation of flux. The value of mmf drops is easily calculated.

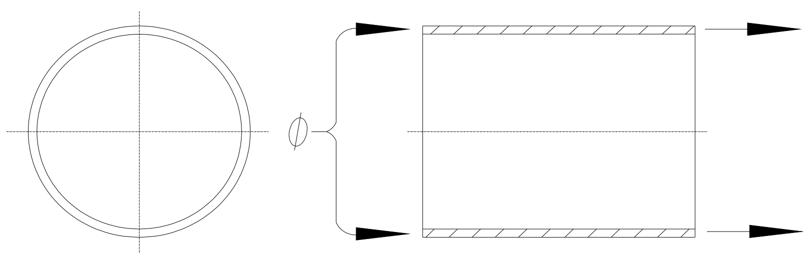

Fig. 3 - Flux Passing Thru a Ferrous Case Tube

The flux density (B) for the case is simply flux (webers) divided by the path area $B = \Phi/A$. From the steel's BH characteristics, H is found from a curve or lookup table or equation. (We will use an equation). Having found H (ampere turns/meter), the mmf drop = H x length of the tube ($\mathcal{F} = H \cdot L$). In a solenoid the flux conducts as a continuous loop following the path of the ferrous components; the loop is not circumferential but rather longitudinal thru the case tube (as shown), radially thru the end flange, across the clearance air gap, plunger, main working air gap, base pole, radially thru the base flange, and to the case again. Thus the flux is virtually the same value thru each component and air gap feature in a series path. At the outset of a magnetic circuit calculation, a single starting flux value for the circuit is set as a completely arbitrary value. By calculating the value of flux density (B) separately for each component and obtaining each corresponding H, the mmf drops for each iron element is computed. The air gap mmf drops are found by $\mathcal{F}_G = \Phi/P_G = \Phi \cdot L/(\mu_0 \cdot A)$ where L and A are the length and area of the gap and $\mu_0 = 4\pi \cdot 10^{-7}$. All mmf drops are summed including those for air gap drops and compared to the coil's mmf. Next, the value of flux is adjusted up or down, as appropriate, and the calculations restarted and reiterated until the summed mmf drops is equal to the coil's ampere turns (which is a constant throughout the calculations) . This convergence of summed drops and coil ampere turns then defines the true solenoid flux relating to the starting conditions of coil ampere turns and plunger position. The solenoid force will be derived from this level of flux in the working air gap and its mmf drop, $\mathcal{F}_G$.

Air Gap Energy and Force: When voltage is initially applied to the coil, the coil current rises exponentially over time (measured in milliseconds) due to inductance and the solenoid force is not instantaneous but rises in concert with the current. The rising coil field begins to align domains in the iron parts which influence neighboring domains and the field of magnetic flux is said to propagate along the iron path and through the air gaps. Since the iron volume has a finite number of domains, an increase in coil mmf (the coil's ampere-turns) will produce a flux level where added coil current produces a diminishing rate of domain alignments and an eventual saturation of the iron. This reduction begins at the observed “knee” as seen on a BH curve followed by a leveling off of flux density (B). Figure 4 illustrates the non-linearity of the BH relationship where B is flux density (webers per square meter) and H is magnetic intensity (ampere turns per meter of length), which will be seen as ampere turns drops, similar to voltage drops across series resistors, across each series steel element in the solenoid based upon its cross sectional area, length and flux density. As seen in the BH curve of figure 4 the iron begins to saturate above the knee of the curve at about B=1.5 and is in hard saturation above

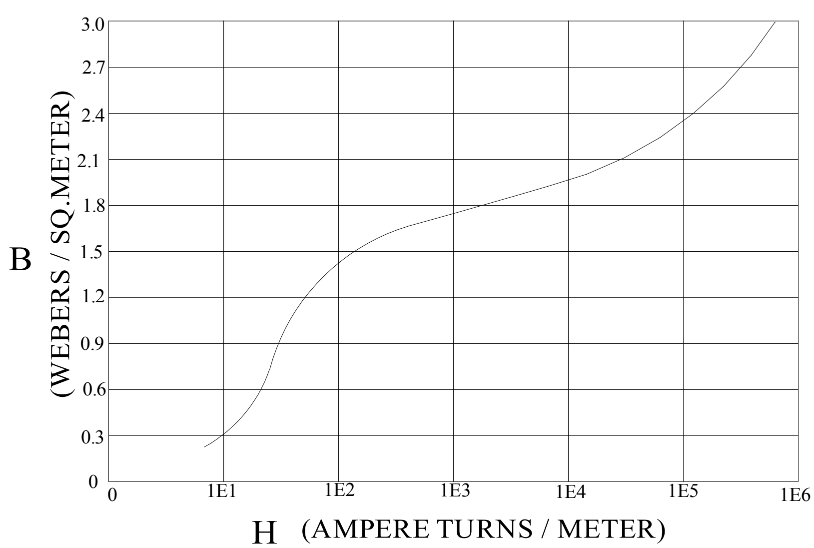

Fig. 4 - BH Curve Representation

about B = 2.2. The iron path within this saturating region begins to create significant drops or losses that limits the mmf (magnetomotive force; ampere-turns) across the working air gap where it is needed to provide the potential energy for performing work. The relationship of B and H is highly non-linear across the usable range of flux densities and, for computer computations, requires a polynomial to define the relationship. Early work before computers required a time-consuming referral to a table or curve of the appropriate steel alloy and many attempts were required to achieve convergence. For this discussion, a typical d.c. solenoid alloy would be no. 1008, or 1215, or 12L14 steel which can be defined by the equation[5]:

$$H = a + bB + cB^2 + dB^3 + eB^4 + fB^5 + gB^6 \qquad (1)$$

where: a= 110.529: b= -1700.2762: c= 14226.658: d= -41673.495:
e= 57423.444: f= -37223.825: g= 9235.1829.

Finally, after the true flux is known, the force between the flat surfaces of the base pole and plunger is given by equation (2):

$$F = B^2 A/2\mu_0 \qquad (2)$$

where $\mu_0 = 4\pi\cdot 10^{-7}$ (the permeability of air), A is a pole face area, B is the flux density in the plunger and F is in newtons.

When the air gap geometry is other than flat faces, the permeability change of the air gap as the plunger moves will require that the force be computed by equation (3) or its equivalent:

$$F = \mathcal{F}_G{}^2/2\cdot dP/ds. \qquad (3)$$

where $\mathcal{F}_G$ is the mmf across the main air gap and dP is the incremental change in air gap permeance during a small increment of plunger displacement, ds.

Recalling that in physics, **work** is the energy transferred to or from an object via the application of force along a displacement[6]; in the work solenoid the product of flux thru an air gap and the mmf drop across the gap determines the energy within the gap. This is the energy well from which mechanical work is derived. Mathematically, when the air gap is allowed to increment towards closure the change in gap energy dw equals the mechanical work F·ds performed by the solenoid. Thus F = dw/ds.

Optimization of Copper and Iron in a Work Solenoid: The foregoing discussion assumes that the solenoid under consideration has been defined as to its geometry externally and internally. But, for new requirements the solenoid designer has the task of determining how much iron volume vs. copper (coil) volume should be allocated for overall efficiency. His goal is to optimize a solenoid design that fits an allowable volume, produces a minimum specified force, stays within power supply limits, has the ability to dissipate its internally generated heat and takes into account (especially when acceleration time is a requirement) the mass of the moved load as well as its own armature mass. Efficiency requires the process of finding the best ratio of copper volume to iron volume for the overall allowable volume of the solenoid. This effort requires the solenoid to operate, under most circumstances, at or near the knee on the BH curve of the iron. Two fundamental factors benefit: (a) The domain alignments of the iron atoms can be viewed as an amplification factor of the coil mmf which peaks at maximum permeability (B/H). (b) Minimum required coil power is achieved resulting in a lower coil temperature rise. Optimization of the copper and iron ratio is more productive than attempting Herculean efforts at winding a precisely perfect coil to gain force efficiency. Note that most solenoid manufacturers

5 Macro Magnetics.pdf mohler 2020
6 Handbook of Physics, 2nd edition, p.2-12 § 5, McGraw Hill publisher.

advertise force data at various power duty cycles; obviously not all can be at an optimum copper and iron ratio as the flux levels must change for each different power level and therefore not operate near the knee of the BH curve for all conditions.

The designer can elect to write a computer routine defining, as variables, the iron and copper elements and incrementally trading copper volume for iron volume until the force peaks. This assumes that the overall volume, coil power and plunger gap (stroke) remain constant. (The alternative is to analyze, via fea, incremental ratios of copper and iron requiring repeated geometry revisions). The computer routine would normally assign a steel cross sectional area for each increment of change which would be applicable for each of the flux carrying elements; coil insulation factors can be held constant, and the remaining volume allocated for copper windings. Either method (fea or macro elements) necessitates a coil winding calculation to accommodate the changing conditions. A method is shown below under figure 5 and is ideal for bobbin wound coils.

Coil Calculation Method: In 1857 J.R. Brown of Brown & Sharpe developed the magnet wire gauge sizes (awg; American Wire Gauge)[7]. Each wire size diameter can be calculated directly from the awg number by equation (4) which can be conveniently implemented into a computer routine.

$$d = 0.46/92^{(awg+3)/39} \qquad (4)$$

Here **d** is in inches. (For awg 0000 use -3 in the exponent, etc). This is the bare copper diameter and must then be modified to include the insulation layer for calculating the coil by equation (5):

$$dI = \mathbf{d}+4.38727E\text{-}03*\mathbf{d}\text{^}0.343976 \quad \text{(Insulated wire diameter)} \qquad (5)$$

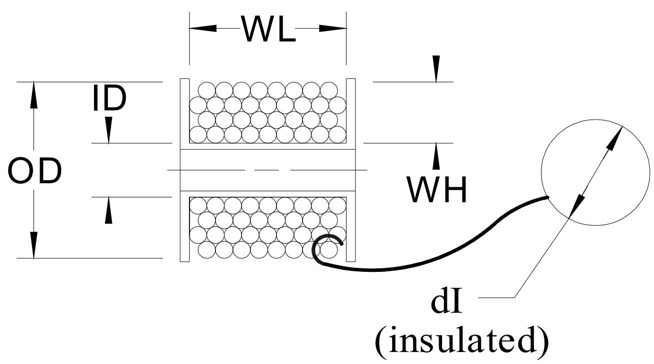

Fig. 5 - Bobbin-Wound Coil

Coil winding computer iteration after finding the insulated wire diameter, **dI**, above (units in inches). This is a pcbasic version:

```
5  User must input od, id, wL, awg
10 pi=3.14159:rad=pi/180
12 d=.46/92^((awg+3)/39):di=d+4.38727E-03*d^.343976
20 wh=(od-id)/2
30 tL=int(wL/di)-1:'turns/layer
40 xwh=di:nL=1:goto 60
50 xwh=xwh+di*cos(30*rad):nL=nL+1
60 if xwh+di*cos(30*rad)<wh then goto 50
80 nt=nL*tL:'total turns
90 R=(xwh+id)*nt*6.787388E-07/(d^2/4):'coil res.@20°C
```

7 Standard Handbook for Electrical Engineers, 8th edition, p.244, A.E. Knowlton, McGraw Hill publisher.

```
100 print nt;R
110 end
```

Note that line 90 for resistance uses the bare copper diameter **d**, not the insulated diameter, **dI**.

Allowable Coil Watts: During the design phase and before the final model can be tested, the heat dissipating ability can be estimated by knowing the overall size of a cylindrical, steel housed solenoid. The computation below is based upon empirical data of solenoids up to 4 inch diameter and anticipates a final stabilized coil temperature of about 105°C when the computed wattage is the cold starting wattage (at about 20°C) and power is on continuously. Mounting the solenoid to a heat sink plate greatly improves the heat dissipation and continuous duty power rating.

Method: For computing the continuous power rating (P), first calculate the total volume (vol) of the solenoid (volume is in cubic inches).

If vol=> 2.06 then P=1.213*vol + 6.0 (watts).
If vol=> 0.186 and vol<2.06 then P=3.772*vol + 0.73 (watts).
If vol<0.186 then P=7.709*vol (watts).

Solenoid inductance is a property that opposes a change of current in the inductor and is dependent upon the number of coil turns, instantaneous current, and circuit flux. It is a calculated or measured result of the defined model and is not used as an input prior to solving the model. The magnetic energy stored in an inductor is given by $w = LI^2/2$ (joules). The inductance value is needful in calculating the current rise time and ultimate acceleration and response time of the solenoid when all other factors are factored in (moving mass, net force vs. position). As mentioned earlier the solenoid force is not instantaneous but ramps upward in concert with the current rise, and, as motion begins the coil current and force are negatively affected by the accelerated motion of the solenoid plunger, which is a function of dynamic inductive reactance. This article does not address the math required to account for the negative going slope of the coil current resulting from the counter emf induced by the accelerating plunger. Before armature motion begins and after it ends, the coil current rise time is dictated by the d.c. resistance of the coil and the inductance of the solenoid by $y = 1 - e^{-tR/L}$. When armature motion occurs a counter emf is generated, based upon rate of motion and instantaneous inductance, and can be equated as an added impedance with that of the coil. This factor, if ignored, accounts for an otherwise erroneously rapid response prediction when electromagnetic theory and newton's laws are employed to compute response times.

Upon calculating a magnetic circuit, the necessary property values are usually also present from which to find the inductance from either of the equations; $L = N\Phi/i$, $L = ƑΦ/i^2$. Note that the inductance value depends upon whether the plunger is fully seated or at some mid stroke position. This applies both for calculating and for measuring the inductance.

The inductance measurement of an iron core solenoid cannot be made with a conventional 1000 hz inductance bridge. The measurement can be made at a low frequency that allows the current and flux to approximate that of the solenoid's continuous rating. Using an autotransformer at 60 hz a.c., adjust the output to the rated voltage, momentarily apply power and record the simultaneous levels of voltage across the coil and current thru the coil. Then calculate the solenoid impedance: $z = e / i$; calculate the solenoid inductive reactance: $X_L = (z^2 - R^2)^{1/2}$; calculate the inductance: $L = X_L / (2\pi \cdot 60)$.

Conclusions: The methods outlined above are not being recommended as a substitute for fea when such software is available but gives some insight into how magnetics problems are solved and were solved in industry before fea became available to the individual designers. Nevertheless, accurate predictions are feasible using even a low level programming language such as Basic and a careful employment of the math.

-end-