# An Informal "Proof" of the "Cross-Over" Multiplicative Method

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## 1 Preamble

First off, I'd like to thank you for taking the time to read this explanation of my newly-discovered method. I'd like to further preface this by saying this original concept was not invented by me–I was actually inspired by Scott Flansburg, in a YouTube video titled "Math Magic Number Secrets Of The Human Calculator By Scott Flansburg;" I can provide a link to said video in the comments below. However, I independently extrapolated upon this idea, and wondered if this would apply to 3x3 digit multiplication, and saw that I could. Then I wondered if I could further do this for any X by X multiplication, and discovered that yes, you could (though at some point this method becomes much harder to do, with all of the numbers you must keep track of, multiply together, and add). I even discovered that you could do uneven digits and decimals (both in the same manner, and uneven in another way as well)! Without further ado, let us get started.

## 2 Preliminary Information

All that is necessary for this "proof" is to know a few concepts. First, you need to know how to truly express numbers in base-10. The number 1,234, for example, is $(1 \cdot 10^3) + (2 \cdot 10^2) + (3 \cdot 10^1) + (4 \cdot 10^0)$. Knowing these numbers are split apart like this allows you to put them on the edge of a square box, delimited by their different place values. This, all on it's own (sans the exponents–they may just be written in standard form, which would be a number like 60), would be the box method that is also commonly taught in schools. The second thing you need to know is that exponents of like bases are added when they are multiplied together. When $10^6$ is multiplied to $10^3$, you add the exponents and get $10^9$. Since we already know that we can express place values in scientific notation, this becomes very important, especially so as the crux of this relies on place value.

## 3 How do you even perform the "Cross-Over" method?

Good question. The cross-over method takes two n-digit numbers and progressively crosses over from the left side to the right side (see Figure 1 and 2 on Page 2), then, once you have reached the right side, you uncross from the left to the right. When you cross over, you add up all the multiplicative crosses that you have made, then carry the remainder and add it to the next addition. It may not be very obvious that this is what is happening when n=2, for example, but you can clearly see this when n=5 or higher.

Table 1: Table of values in a worked out box method of two n-digit numbers ab...cde and vw...xyz. The corresponding legend is below.

| | $a \cdot 10^{n-1}$ | $b \cdot 10^{n-2}$ | ... | $c \cdot 10^2$ | $d \cdot 10^1$ | $e \cdot 10^0$ |
|---|---|---|---|---|---|---|
| $v \cdot 10^{n-1}$ | $\upsilon$ | $\chi$ | ... | $\xi$ | $\nu$ | $\mu$ |
| $w \cdot 10^{n-2}$ | $\chi$ | $\phi$ | ... | $\nu$ | $\mu$ | $\lambda$ |
| ... | ... | ... | ... | ... | ... | .. |
| $x \cdot 10^2$ | $\xi$ | $\nu$ | ... | $\epsilon$ | $\delta$ | $\gamma$ |
| $y \cdot 10^1$ | $\nu$ | $\mu$ | ... | $\delta$ | $\gamma$ | $\beta$ |
| $z \cdot 10^0$ | $\mu$ | $\lambda$ | ... | $\gamma$ | $\beta$ | $\alpha$ |

| LEGEND: | | |
|---|---|---|
| $\alpha - 10^0$ | $\lambda - 10^{n-2}$ | $\phi - 10^{2n-4}$ |
| $\beta - 10^1$ | $\mu - 10^{n-1}$ | $\chi - 10^{2n-3}$ |
| $\gamma - 10^2$ | $\nu - 10^n$ | $\upsilon - 10^{2n-2}$ |
| $\delta - 10^3$ | $\xi - 10^{n+1}$ | |
| $\epsilon - 10^4$ | | |

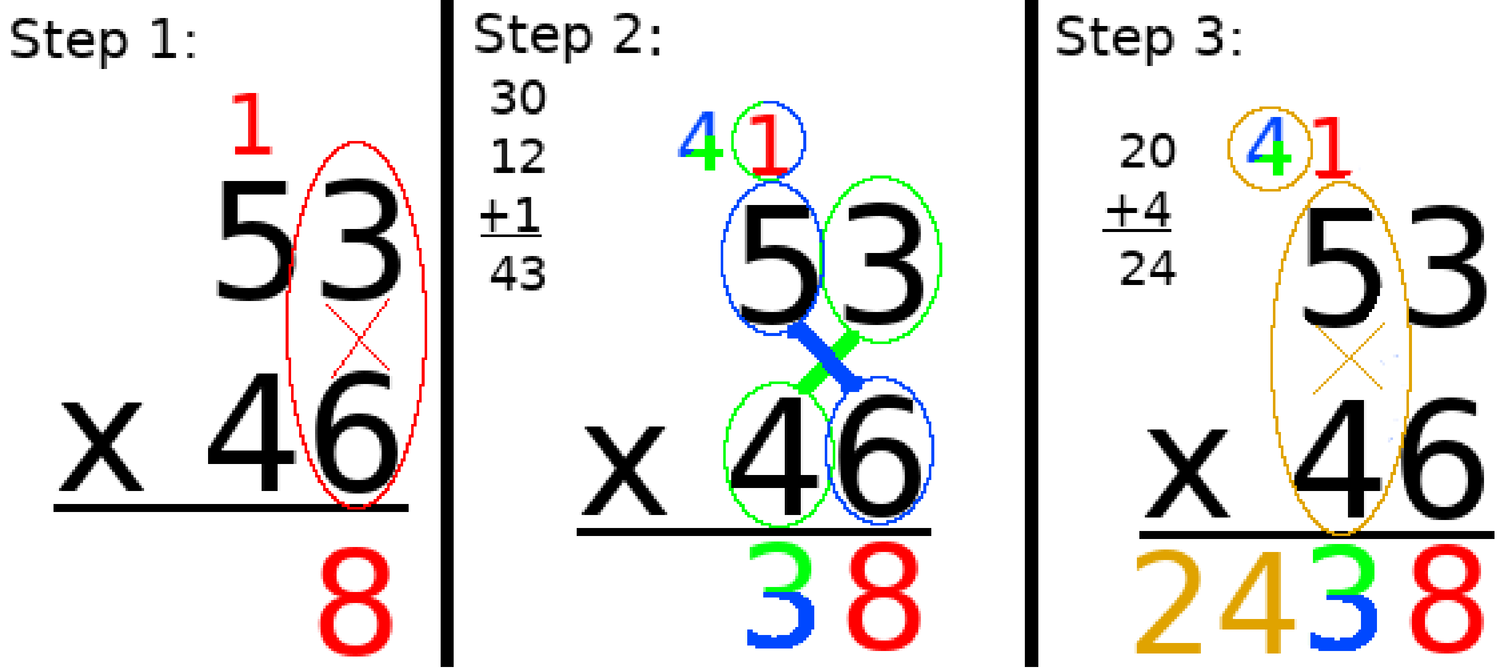


Figure 1: A worked out example for the "cross-over" method.

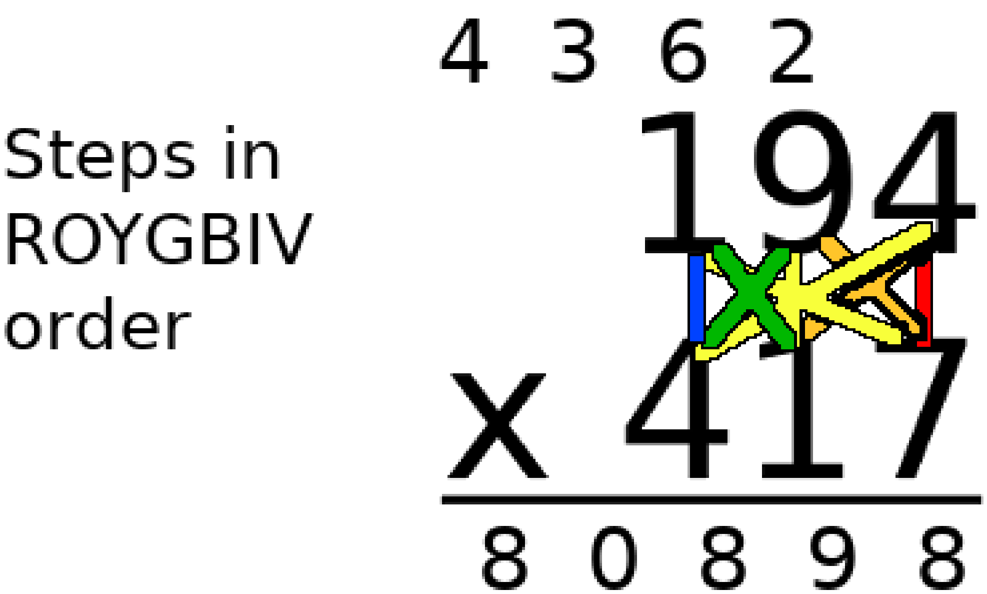


Figure 2: A less worked out example of a 3x3 number.

# 4 Why does this work?

As I previously mentioned in "Preliminary Information," this entirely relies upon the ability to put individual digits next to their power of ten (place value) and the additive properties of these powers of ten. The table shows how you can only get the ones place value in one square, for example. You can get the tens place in two places, and so on and so forth. You are able to get the n-1 place in n places, and after that the number of these values had recedes back towards the top, the 2n-2 place. The occurance of these place values is actually quite important, because you are effectively adding up the amount of that place value when you cross over and add the results. For example, adding (4x5) and (3x2) would yield 26, so if we had 26 tens, that is actually 2 hundreds and 6 tens left over. That is why you carry and add these carries, since you are determining the place value of the number instantaneously. This can become quite memory intensive with larger numbers, so while more efficient and neater for smaller numbers, it may be worth it to use another method for larger numbers.

# 5 How do uneven digit numbers and decimals work?

Well, both of these problems can be solved in one way! Let's say our first number is c, and our second number is d. Therefore, the product must be cd. However, supposing that c is one digit shorter than d, we can even these out. If we multiply c by 10, we know that these place values will shift one to the left and leave you with a zero at the end of the number. Also, (10*c)d is 10cd by the associative property. So, we evened out the digits and got an answer, but in order to get the true answer, you must divide by 10. Luckily for us, we work in base 10, so dividing by 10 is as easy as multiplying was: the decimal is at the end of a whole number, so you move it back as many digits as you added or multiplied by 10 to get our original cd. Decimals work in the same way, except that instead of moving the entire number one to the left and tacking on a zero to the end, you move the decimal one place to the right. With this combination of cancelling decimals and evening out numbers, any two numbers can be multiplied together.

Another way you could adjust the number of digits in two uneven numbers is by adding zeros to the end. Since adding zeros to the beginning of a number does not change the value, this is perfectly valid, but this will not work on decimals, so a catchall would be to always multiply by as many 10s as you may need. However, adding just zeros to the front saves time by avoiding the necessity of dividing in the end, and may be more efficient in the long run. You may also divide numbers that end in zero to even numbers out and get them in the form X by X. You will just have to multiply by 10 in the end, as you would have .1cd.

# 6 Drawbacks

As I've mentioned a few times throughout, this method will become very inefficient with very large numbers, especially if you are doing the multiplication and subsequent addition mentally. With the regular method, you do one multiplication, write down the product, and save all summation until the very end of the process in a rigid but airtight procedure. The method is also inefficient with numbers with large differences between their sizes. There would be unnecessary but avoidable steps in doing this, such as many multiplications by zero that can be discarded, but it would work.

# 7 What next?

The next thing I wanted to do with this process is find an efficient way to do this backwards. This way, you could immediately get a better estimate of the magnitude of the number, such as if you are doing this mentally, but so far I have not come up with a solution that deals with the potential awkward carrying and the problems that would arise as carries changed place values, which effect other place values in a ripple effect of sorts.

I just wanted to say thank you again for reading this through in its entirety, and to please give me any comments, questions, or criticisms of my first informal "proof" of this method and how and why it works.